Product Cycles and Phillips Curve:

Micro Analysis

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Product Cycles and Phillips Curve: Micro Analysis *

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Abstract

We estimate a model explicitly describing product entry and exit in a frictional product market for micro category data, i.e., snack foods data. The paper shows that the first/new price when a product entry mainly determines the average price and search frictions quantitatively contribute to price variations. A price change probability by the product entry is 8.4 percent though a price change probability after product entry is only 1.4 percent. Search frictions increase the price variation by 50.5 percent. These results demonstrate that fundamental elements in explaining price variations are ignored in the official price data, such as the consumer price index and the producer price index, and conventional models that exclude product entry and exit. Endogenous product entry inducing extensive margins and search frictions are quantitatively important in analyzing prices.

Keywords: Phillips curve; product and price cycles; search and matching

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1 Introduction

Using product-level data, we show that product entry and exit hold a non-trivial effect on price dynamics. We use data on snack foods to maintain homogeneity across products and estimate the Phillips curve developed by Dong et. al (2024). This model explicitly describes product entry and exit in a frictional product market. By limiting the data, we evaluate the roles of product entry and exit in prices regardless of product heterogeneity.

The estimation shows that the first/new price when a product entry mainly determines the average price for snack foods. A price change probability by the product entry is 8.4 percent though a price change probability after product entry is only 1.4 percent. Moreover, the search friction works well in a product market and significantly affects the average price. The search friction constraints unmatched products to find a retailer with only a 55 percent probability. Shocks related to search frictions, the matching efficiency shock and the free entry shock, play important roles and explain 15.7 and 9.67 percent of price variations, respectively. Here, a demand shock is dominant to prices and explains 71.71 percent of price variations.

Search frictions quantitatively contribute to price variations. When we use a model excluding search frictions under the estimated parameters, the price variation decreases by 49.5 percent for the aggregate price. These results demonstrate that endogenous product entry and extensive margins are fundamental elements in explaining price variations.

Even when we break down the data into the top two companies and small companies in sales share, search frictions exist for these companies. Interestingly, for the top two companies, a cost-push shock explains 69.63 percent of price variations. On the other hand, a demand shock explains 77.53 percent of price variations for small companies. It is consistent with the data in which the first price average is lower (higher) than the average price for the top two companies (for small companies). The top two companies can change prices to a positive cost-push shock under a positive inflation rate in the sample period and small companies can not add the cost well to their prices.

Our paper contrasts Dong et. al (2024) that estimate the Phillips curve describing product entry and exit for all products in Japanese supermarkets. Their analysis is as comprehensive as the one using the official consumer price index. However, different product natures, such as chocolates and snacks, are mixed in the estimation. We complement their analysis by focusing on a more detailed category of products, i.e., snack foods, and in a narrow region, i.e., Tokyo metropolitan area, to reconfirm well fit of the model to the data.

The rest of our paper is organized as follows. Section 2 describes our data and documents a set of observed facts. Section 3 introduces the model with product and price cycles and shows the generalized version of the New Keynesian Phillips. Section 4 contains the Bayesian estimation of our model and the results. In Section 5, we quantitatively evaluate the effects of endogenous product entry and extensive margins on prices. In Section 6, we show robust analysis. Finally, Section 7 concludes.

2 Micro Observations for Snack Foods

We use the Nikkei POS scanner data.[1] Our data includes sales prices and quantities for each product at each retail shop on each day from January 2005 to March 2021. To focus on homogeneous products, our analysis is particularly restricted to snack foods and the 25 supermarkets in Tokyo metropolitan area.[2] These supermarkets appear throughout the sample to avoid any bias caused by shop bankruptcy, i.e., a shop cycle, in price, entry rate, and exit rate. We interpret the data from the 25 supermarkets as a random sample to observe product and price cycles at the product level. Our data includes 723 snack foods as the maximum and 407 as the minimum in the sample periods.[3] About 90 companies produce snack foods in 2019 year.

An advantage of our data is that we can observe product cycles. An individual product's life cycle can be identified through its entry into and exit from the product

[1] Appendix A shows detailed information about the data, including product identification by Japanese Article Number (JAN) code, average price, entry rate, and exit rate.

[2] Tokyo metropolitan area includes four prefectures, Tokyo, Kanagawa, Chiba, and Saitama.

[3] See Appendix A for a description of how we convert daily data into monthly data.

market. We can also observe price cycles, by which we refer to how the individual product's price changes during its life cycle. The price information provides evidence about price-setting behavior.

We summarize our key empirical facts as four observations. We will use these observations to guide our theoretical model and then estimate it using Bayesian inferences.

Observation 1: an average price for snack foods.

Based on individual prices in the Nikkei POS scanner data, we construct the sales-weighted average price across products and shops including temporary sales for each month. An average of the average price for snack foods is 122.26 yen and its standard deviation is 5.75 yen for the sample period.[4]

Observation 2: product cycles.

The JAN (Japanese Article Number) codes in Nikkei POS scanner data allow us to identify the numbers of new products and exiting products, which are used to calculate product entry and exit rates. Table 1 shows basic statistics of monthly product entry and exit. The average of the number of products is 572.9 and the standard deviation of the number of products is 61.6, as shown in Table 1. The number of products changes over time.

The product entry rate is calculated as the number of newly introduced products by producers in a given quarter divided by the total number of products in that quarter. We identify a new product when the new product appears in at least one retail shop. The product exit rate is calculated as the number of exiting products in a given quarter divided by the total number of products in that month. We classify an existing product as exiting from the market when none of the retail shops sell the product. Note that we define entry rates and exit rates at the product level and not at the shop level.[5] The average product entry rate of snack foods is 0.086 and its standard deviation is 0.025 as

[4]See Appendix A for details to calculate the average price.

[5]See more details of the definitions of the entry rate and the exit rate in Appendix A.

shown in Table 1. The average product exit rate is 0.084 and its standard deviation is 0.021. The product entry rate is more volatile than the product exit rate. The product exit rate implies that the product cycle is on average 12 months. We calculate the duration of a product by taking the inverse of the product exit rate.

Observation 3: price cycles.

We find that the first/new prices behave differently from the subsequent prices. As shown in Table 1, the standard deviation of the average price is 5.77 yen and it is 23.4 yen for the average new price. It implies that existing prices are not as flexible as new prices. New prices have a larger effect on the average price than existing prices do.

Observation 4: search frictions.

We provide evidence on search frictions by using the Nikkei POS data to construct matching ratios for retailers. In the retailer's matching ratio for a given product, the numerator is the number of retail shops selling this product in a given period. The denominator is given by the number of shops in the sample, i.e., 25 supermarkets. In addition to the simple average of the matching ratio, we calculate the weighted average of the matching ratio by sales weights.[6]

The average matching ratios are 0.3 for the simple average and 0.69 for the weighted average by sales at a monthly frequency. Matching ratios are sufficiently less than one. This observation implies the existence of search frictions in the snack foods market.

3 Model with Product and Price Cycles

We basically use the model developed by Dong et. al (2024). The model captures product entry and exit with a frictional product market.

[6] In the case of a simple average, the matching ratio is given by the average of the matching ratio with equal weights across products. Appendix A has a detailed description of how we calculate these matching ratios.

3.1 Model Setting

Time is discrete and continues forever. A measure 1 of retailers searches for products in a decentralized product market. An endogenous measure of products also searches for retailers. Each product in the market represents a distinct variety. Products can enter the product market at a cost κ_t that works as an entry shock.

Let the measure of unmatched retailers be u_t at time t and the measure of products searching for retailers be v_t. The matching function exhibits constant return to scales and is given by

$$m\left(u_t, v_t\right) = \chi_t \frac{u_t v_t}{\left(u_t^\gamma + v_t^\gamma\right)^{\frac{1}{\gamma}}} \qquad \text{with } \gamma > 0,$$

where χ_t is a matching efficiency shock and it is one in steady state. Define market tightness as $\theta_t = v_t/u_t$. The probability for a product to be matched with a retailer is denoted as $s(\theta_t)$ and the probability for an unmatched retailer to find an available product is denoted as $q(\theta_t)$, where

$$s(\theta_t) = \frac{m_t}{v_t} = \chi_t \frac{1}{\left(1+\theta_t^\gamma\right)^{\frac{1}{\gamma}}}, \tag{1}$$

$$q(\theta_t) = \frac{m_t}{u_t} = \chi_t \frac{\theta_t}{\left(1+\theta_t^\gamma\right)^{\frac{1}{\gamma}}}. \tag{2}$$

We assume that $s(0) = 1$ and $q(\infty) = 1$. To simplify notation, we use (s_t, q_t) directly and omit the argument θ_t when there is no confusion. Each match is destroyed at the end of a period with an exogenous probability $\rho \in (0, 1)$.

Once a retailer and a product match, they exchange Z units of the product and negotiate a new price through the Nash bargaining protocol. With a probability α, there is no renegotiation of the price within the match, and the price does not change. With a probability $1 - \alpha$, the match can renegotiate the price. New prices are negotiated when either new matches are formed or the match gets the opportunity to renegotiate. This infrequent negotiation of prices directly follows Shimer (2004), Hall (2005), and Gertler and Trigari (2009) in labor search models. For simplicity, the amount of production in each match Z is exogenous. Moreover, the cost of producing Z units of any product is X_t, where X_t can include any cost of production even though we do not specify the production function at this stage. Changes in X_t could be interpreted as potential cost-push shocks. The benefit for retailers of acquiring Z units of any product is given by Z_t^B, where Z_t^B is a random shock and depends on the sales revenue of the final good. We view Z_t^B as a demand shock to products.

The free entry condition for a product is

$$\kappa_t = \beta s_t \mathbb{E}_t J_{t+1}\left(\tilde{P}_{t+1}\right), \tag{3}$$

where $\mathbb{E}_t$ is the expectations operator. The value for a product with a newly negotiated price $\tilde{P}_{t+1}$ is $J_{t+1}(\tilde{P}_{t+1})$. Since we assume that each match entails a distinct product, this free entry condition decides the number of new products in the product market. A product enters the market when the gain from selling the new product is enough to cover the cost of entry. If a product is matched with a retailer, production and trade will occur in the following period. There is a one-period lag for production after a new match, similar to the timeline of Gertler and Trigari (2009).

The value function for a newly matched product at time t is

$$J_t(\tilde{P}_t) = Z\tilde{P}_t - X_t + \beta\left(1-\rho\right)\mathbb{E}_t\left[\alpha J_{t+1}\left(\tilde{P}_t\right) + (1-\alpha)J_{t+1}\left(\tilde{P}_{t+1}\right)\right]. \tag{4}$$

If the match cannot renegotiate, the price unchanges. If the match renegotiates, a new price $\tilde{P}_{t+1}$ will be determined. The term $Z\tilde{P}_t - X_t$ is the flow benefit of being in a match, and $\beta\left(1-\rho\right)\mathbb{E}_t\left[\alpha J_{t+1}\left(\tilde{P}_t\right) + (1-\alpha)J_{t+1}\left(\tilde{P}_{t+1}\right)\right]$ shows the continuation value of the match.

Now consider the value functions for a retailer. Let $V_t^1\left(\tilde{P}_t\right)$ denote the value function for a matched retailer with a newly negotiated price $\tilde{P}_t$ at time t and

$$V_t^1\left(\tilde{P}_t\right) = Z_t^B - Z\tilde{P}_t + \beta\mathbb{E}_t\left[\alpha\left(1-\rho\right)V_{t+1}^1\left(\tilde{P}_t\right) + (1-\alpha)(1-\rho)V_{t+1}^1\left(\tilde{P}_{t+1}\right) + \rho V_{t+1}^0\right]. \tag{5}$$

The flow benefit of the match is given by the term $Z_t^B - Z\tilde{P}_t$. If the match survives at time $t+1$, the continuation value is $V_{t+1}^1\left(\tilde{P}_t\right)$ with a probability α. With a probability $1-\alpha$, the match renegotiates a new price $\tilde{P}_{t+1}$ and the continuation value is $V_{t+1}^1\left(\tilde{P}_{t+1}\right)$. If the match is destroyed, the retailer becomes unmatched with the value function V_{t+1}^0.

The value of an unmatched retailer is

$$V_t^0 = \beta \mathbb{E}_t \left[q_t V_{t+1}^1 \left(\tilde{P}_{t+1} \right) + (1 - q_t) V_{t+1}^0 \right]. \tag{6}$$

The unmatched retailer finds a match with a probability q_t. Production will take place in the following period, and the value of the match is therefore $\mathbb{E}_t V_{t+1}^1 \left(\tilde{P}_{t+1} \right)$. With the complementary probability $1 - q_t$, the unmatched retailer remains unmatched and has the continuation value V_{t+1}^0. Here, the benefit from having a new match is $V_t^1 \left(\tilde{P}_t \right) - V_t^0$ for the retailer.

The Nash bargaining solves the price $\tilde{P}_t$ in

$$\max_{\tilde{P}_t} \left[V_t \left(\tilde{P}_t \right) - V_t^0 \right]^{1-b} \left[J_t^1 \left(\tilde{P}_t \right) \right]^b,$$

where b is the bargaining power for the producer. The solution $\tilde{P}_t$ is determined by

$$b \left[V_t \left(\tilde{P}_t \right) - V_t^0 \right] = (1 - b) J_t \left(\tilde{P}_t \right). \tag{7}$$

Lastly, we describe the flow conditions and the aggregate price index. The measure of unmatched retailers in the beginning of period t is

$$u_t = 1 - N_t, \tag{8}$$

where N_t denotes the measure of matches. It evolves according to

$$N_{t+1} = (1 - \rho) N_t + q_t u_t. \tag{9}$$

As prices in the new matches are set through Nash bargaining and the old prices in survived matches are either not adjusted from time t to time $t+1$ or replaced by newly negotiated prices, we use an aggregate price index P_t to denote the aggregate price in the economy at time t:

$$N_t P_t = (1 - \rho) \alpha N_{t-1} P_{t-1} + (1 - \rho)(1 - \alpha) N_{t-1} \tilde{P}_t + q_{t-1} u_{t-1} \tilde{P}_t. \tag{10}$$

The aggregate price index completes the description of the model, where (1), (2), (3), (4), (5), (6), (7), (8), (9), and (10) are used to solve the model.[7]

[7]See Appendix C for more details of this model.

In our model, entry decisions are endogenous and depend on the parameters and shocks. However, we treat exits as exogenous. The model generates an endogenous number of products and can be used to examine how product entry is correlated with price and demand.

For the aggregate price index, the model has both extensive and intensive margins of price changes. The extensive margins refer to the change in the composition of products in the aggregate price index. The intensive margins refer to the price change by each product. The endogenous numbers of existing and new matches give rise to endogenous extensive margins of price changes. The newly negotiated prices reflect endogenous intensive margins and the price discounting factor produces an exogenous intensive margin of price changes.

3.2 The New Keynesian Phillips Curve

We highlight the role of an endogenous number of products with search frictions by log-linearizing the system of equations around a constant steady state with zero inflation. Linearized price equations are convenient for revealing the features of price dynamics, in particularly compared with the standard New Keynesian Phillips curve by Calvo (1983) and Yun (1996). We express the log-deviation of a variable (e.g., P_t) from its steady state value (P) by placing a hat ($\hat{}$) over the symbol ($\hat{P}_t$).[8]

We have the following linearized price equation:

$$\begin{aligned} \pi_t &= \alpha\beta(1-\rho)\mathbb{E}_t\pi_{t+1} \\ &- b\beta q(V^1 - V^0)\frac{[1-\alpha\beta(1-\rho)]\left[1-\alpha\left(1-\rho\right)\right]}{Z\tilde{P}}\left(\hat{\theta}_t + \hat{\kappa}_t\right) \\ &+ b\frac{[1-\alpha\beta(1-\rho)]\left[1-\alpha\left(1-\rho\right)\right]Z^B}{Z\tilde{P}}\hat{Z}_t^B \\ &+ (1-b)\frac{[1-\alpha\beta(1-\rho)]\left[1-\alpha\left(1-\rho\right)\right]X}{Z\tilde{P}}\hat{X}_t, \end{aligned} \tag{11}$$

where the inflation rate is defined as $\pi_t \equiv \hat{P}_t - \alpha(1-\rho)\hat{P}_{t-1}$.

[8]See details for the steady state in Appendix B and a complete set of the linearized model in Appendix C.

This Phillips curve includes three types of shocks: a free entry shock, a final demand shock, and a cost-push shock in production. The free entry shock is a new shock for the Phillips curve and is specific to our model with endogenous entry of products. Current inflation depends on the expected future inflation, the number of products, the number of new products, the market tightness, the free entry shock, the demand shock, and the cost-push shock. The probability of price adjustment α and the exit rate ρ affect current inflation. The market tightness, measured by the ratio of available products to the unmatched retailers, is negatively related to inflation. The exit rate ρ, the steady state matching probability q, and the bargaining power of producers b affect the response of the inflation rate to market tightness.

Product market frictions have explicit effects on inflation through the market tightness $\hat{\theta}_t$, which links product cycles with price dynamics. In response to changes in the demand for products, the entry rate of products changes. Therefore, the number of new matches M adjusts, which implies changes in the fraction of products with new prices. Changes in the number of new matches will affect the number of total matches N in the following period. As a result, the number of existing matches that either adjust prices by the price discounting/premium factor or have the opportunity to set new prices changes. One way to interpret our results is that the model endogenizes the price change probability, such as the Calvo parameter, through a search and matching product market. The response of inflation to demand shocks increases as α decreases due to an intensive margin effect.

4 Estimation and Results

We estimate the model with Bayesian inferences using Dynare 5.2. For estimation, we log-linearize the system of equations around a constant steady state with zero inflation.

4.1 Data and Observation Errors

We use Nikkei POS scanner data at a monthly frequency over the sample from 2005M1 to 2021M3. The estimation uses data from three time series: the average price, the number of products, and the final sales of new products, which correspond to P_t, N_t, and Z_t^B in our model. All these data are percentage deviations from trends calculated by the HP filter.

To map data into our model, we assume observation errors in the average price, the number of products, and the sales of the new products to adjust the gap between the data and the model. For the number of products, the observation error is given by 10 percent of one standard deviation of the number of products in the data, i.e., 0.05/10. Similarly, the observation error for the final sales of the new products is 0.35/10. Observation errors are assumed to be i.i.d. normally distributed with zero mean.

The gap between P_t and the average price can come from two sources. Firstly, the average price is one between retailers and consumers. Our model focuses on prices between retailers and producers. Secondly, the average price includes temporary sales. Our model does not incorporate temporary sales at store levels.

We justify the usage of the average price by showing retail prices measured by the CPI closely depend on producer prices measured by the CGPI (the Corporate Goods Price Index) after excluding temporary sales in Japan. Here, CPI corresponds to prices between consumers and retailers, and CGPI corresponds to prices between producers and retailers. CPI excludes temporary sales prices, and the numbers of goods in baskets are constant in the CPI and CGPI. We limit these data to correspond to product categories in the Nikkei data and calculate their correlation. The correlation is quite high and 0.88 at the quarterly frequency in our sample period. So the average price includes price information between producers and retailers. We use the average price for estimation and assume ϵ_t^{err} to fill the gap between the average price and P_t from the model. The standard deviation of observation errors is estimated by Bayesian inference as follows.

$$\hat{P}_t^{obs} = \hat{P}_t + \epsilon_t^{err}, \tag{12}$$

where $\hat{P}_t^{obs}$ is the SBPI, $\hat{P}_t$ is the trend component from the model's P_t, and ϵ_t^{err} is i.i.d. normally distributed with zero mean and a standard deviation σ^{err}. Here, ϵ_t^{err} absorbs unnecessary price variations through temporary sales. We can interpret this equation as retailers ex post setting retail prices by a constant markup on producer prices with temporary adjustments.

4.2 Calibrated Parameters and Shocks

We set the value of the discount factor as a conventional value of 0.9967 for a monthly model. For markups between producers and retailers $Z\tilde{P}/X$ and between retailers and consumers $Z^B/Z\tilde{P}$ in Japan, we follow Kondo (2020) and assume 1.687. This is a value-added markup and is consistent with our model. We calibrate the exit rate from data as $\rho = 0.084$. Steady state values of Z and $\tilde{P}$ normalized to ones.

We consider four types of shocks: the matching efficiency shock χ_t, the free entry shock κ_t, the final demand shock Z_t^B, and the cost-push shock X_t, as shown in a model. Each shock follows an AR(1) process where

$$\hat{\chi}_t = \rho_e \hat{\chi}_{t-1} + \epsilon_t^e,$$

$$\hat{\kappa}_t = \rho_{fe} \hat{\kappa}_{t-1} + \epsilon_t^{fe},$$

$$\hat{X}_t = \rho_x \hat{X}_{t-1} + \epsilon_t^x,$$

$$\hat{Z}_t^B = \rho_{zb} \hat{Z}_{t-1}^B + \epsilon_t^{zb}.$$

The disturbances $(\epsilon_t^e, \epsilon_t^{fe}, \epsilon_t^x, \epsilon_t^{zb})$ are i.i.d normally distributed with mean zero and standard deviations $(\sigma_e, \sigma_{fe}, \sigma_x, \sigma_{zb})$, respectively.

4.3 Prior Distributions

Table 2 shows moments for the prior distributions of the structural parameters. We follow the setting in Dong et. al (2024) that use all products from the Nikkei POS data. We assume that the matching friction parameter γ follows an Inverse Gamma Distribution since it should be positive. The producer's bargaining power b needs to be in [0, 1], so we assume that it follows a Beta distribution. Regarding the shock persistent parameters and the standard deviations of the disturbances, we follow previous studies and assume Beta distributions and Inverse Gamma Distributions, respectively. For the means of the standard deviations of the demand shocks and the cost-push shock, we set 0.2. For the mean of the standard deviation of price observation errors, we assume 50 percent of the standard deviation of the average price, i.e., 0.038/2. The means of the standard deviations of the matching efficiency shock and the free entry shock are assumed to be 0.4.

4.4 Posterior Distributions and Variance Decomposition

Table 3 shows moments for the posterior distributions. The estimation results suggest the existence of search frictions among products. The mean of the matching curvature parameter γ is 0.855. We can calculate the steady state matching probability for an unmatched product as 0.546.[9] Only about 55 percent of unmatched products can find a retailer. We also provide results on the role of matching frictions in the product market through the variance decomposition in Table 4.

In Table 3, the bargaining power parameter for the product is 0.747. The optimal price change probability after product entry $1 - \alpha$ is 0.014. It implies that only 1.4 percent of products change prices each month after the first price. From product entry, 8.4 percent of products set the first price every month on average since the entry rate and the exit rate are the same in the steady state. Our result suggests that price changes are mainly brought by product entry.

As shown in Table 4, shocks related to search frictions, the matching efficiency shock and the free entry shock, play important roles and explain 15.7 and 9.67 percent of price variations, respectively. On the other hand, conventional shocks for prices, the demand shock and the cost-push shock, explain 71.71 and 2.92 percent of price variations, respectively. Thus, the demand shock has the largest effect on the snack prices and about

[9] Using the estimated parameters in Table 3, we can calculate all the steady state values such as $\kappa = 2.55$, $q = 0.346$, $s = 0.546$, $N = 0.805$, $m = 0.068$, $u = 0.195$, $v = 0.124$, and $\theta = 0.633$.

25 percent of price variation is explained by shocks related to search frictions.

5 Evaluating Endogenous Product Entry and Extensive Margins

In this section, we use an estimated model to evaluate how much price variation is explained by endogenous product entry and the extensive margins.

5.1 Exogenous Entry Model

To quantitatively investigate the role of endogenous entry, we shut down endogenous entry by assuming exogenous entry and exit rates such that the numbers of new products and total products are constant. This implies that the fraction of products with a new price is also constant. We label this model an exogenous entry model and check its quantitative performance.

Without the free entry condition, we modify the value function for a newly matched product as

$$\bar{J}_t^1(\tilde{P}_t) = Z\tilde{P}_t - X_t + \beta \mathrm{E}_t \left[\alpha(1-\rho)\bar{J}_{t+1}^1(\tilde{P}_t) + (1-\alpha)(1-\rho)\bar{J}_{t+1}^1(\tilde{P}_{t+1}) + \rho \bar{J}_{t+1}^0 \right].$$

On the other hand, the value of a product without a match is

$$\bar{J}_t^0 = \beta \mathrm{E}_t \left[\bar{s}\bar{J}_{t+1}^1(\tilde{P}_{t+1}) + (1-\bar{s})\bar{J}_{t+1}^0 \right],$$

where we have $\bar{q} = \bar{s}$ represent steady state matching probabilities. The measure of unmatched products is given by

$$v_t = 1 - N_t.$$

In this case, the model is linear since variables related to product market frictions and matches, such as q_t, s_t, N_t, u_t, v_t, and θ_t, are constant. This model produces very similar inflation dynamics to demand shocks and cost-push shocks to the ones generated by the New Keynesian Phillips curve as

$$\begin{aligned}\pi_t &= \alpha\beta(1-\rho)\mathbb{E}_t\pi_{t+1} \\ &+ b\frac{[1-\alpha\beta(1-\rho)]\,[1-\alpha\,(1-\rho)]\,Z^B}{Z\tilde{P}^e}\hat{Z}_t^B \\ &+ (1-b)\frac{[1-\alpha\beta(1-\rho)]\,[1-\alpha\,(1-\rho)]\,X}{Z\tilde{P}^e}\hat{X}_t. \end{aligned} \tag{13}$$

The new price is given by $\hat{\tilde{P}}_t = [1-\alpha\,(1-\rho)]\,\pi_t$ and $\tilde{P}^e$ is the steady state of the new price. The effects of product and price cycles appear through ρ and α.

5.2 Endogenous Entry Effect

We compare the models with endogenous product entry and exogenous product entry by simulating the models' responses to the four types of shocks. The endogenous entry model is the baseline model, as in Section 3. For the exogenous entry model, (13) describes the intensive margin effect. Current inflation only depends on expected future inflation, the demand shock and the supply shock. The coefficients do not include steady state variables except for X, Z^B, Z and $\tilde{P}^e$. Using the estimated parameters in Table 3 and setting $Z = 1$ and $Z^B = 1.687$, the steady state new price $\tilde{P}^e$ is calculated as 1.41. This steady state new price depends on the estimated parameters and does not depend on other steady state values. Note that owing to the exogenous entry and exit rates, only the demand and supply shocks out of the four shocks affect inflation dynamics.

Table 5 shows the standard deviations of prices from our simulations. The standard deviation of the new price $\tilde{P}_t$ increases by 51.9 percent due to endogenous product entry when we compare the baseline model and the exogenous entry model. This is given by an increased intensive margins by endogenous product entry.

For the aggregate price P_t, the price variation increases by 50.5 percent when comparing the endogenous entry model with the exogenous entry model. These results demonstrate that endogenous product entry and extensive margins are fundamental elements in explaining price variations.

6 Robust Analysis

In this section, we restrict samples and show that our estimation result in the last section is robust. In particular, we show two cases. The first one is to use snack foods data for two top sales companies that have 50 percent shares in the snack market. The second one is to restrict data only to below the top 10 companies.

6.1 Top Companies

In our data for snack foods, we have about 90 companies that produce snack foods. Among these companies, the top two companies, Calbee and KOIKE-YA Inc, have about 50 percent sales share in 2019. They produce about 440 products, about 36 percent of all snack foods.

We restrict data to these two top companies to secure homogeneity across products. We calibrate the exit rate from data as $\rho = 0.091$. For the number of products, the observation error is given by 10 percent of one standard deviation of the number of products in the data, i.e., 0.069/10. Similarly, the observation error for the final sales of the new products is 0.42/10. Observation errors are assumed to be i.i.d. normally distributed with zero mean. For the mean of the standard deviation of price observation errors, we assume 50 percent of the standard deviation of the average price, i.e., 0.047/2. Other settings for the estimation remain the same as in Table 2.

Table 6 shows moments for the posterior distributions. The estimation results suggest the existence of search frictions among products by the top two companies. The mean of the matching curvature parameter γ is 0.745. We can calculate the steady state matching probability for an unmatched product as 0.499.[10] Only about 50 percent of unmatched products can find a retailer.

As shown in Table 6, the bargaining power parameter for the product is 0.711. The optimal price change probability after product entry $1 - \alpha$ is 0.023. It implies that only

[10] Using the estimated parameters in Table 6, we can calculate all the steady state values such as $\kappa = 2.15$, $q = 0.297$, $s = 0.499$, $N = 0.765$, $m = 0.07$, $u = 0.235$, $v = 0.14$, and $\theta = 0.594$.

2.3 percent of products change prices each month after the first price. This number is slightly bigger than the one from Table 3. However, our result suggests that price changes are mainly brought by product entry since companies can set a new price with a probability of 9.1 percent.

As shown in Table 7, for the top two companies, the cost-push shock explains 69.63 percent of price variations and it is the largest contribution among the four types of shocks. This is a clear difference from Table 4. It implies that it is not so difficult to add production costs to their prices for companies with a large sales share under a positive inflation rate in our sample period. We reconfirm it in the data. The average price of 123.96 yen is higher than the average new price of 111.99 yen for the top two companies. These companies can increase prices after product entries according to production costs.

6.2 Small Companies

We focus on small companies in comparison to the top two companies. We define small companies as being below the top 10 companies in sales in 2019. These small companies have about 15 percent sales share in 2019. They produce about 30 percent of all snack foods.

We calibrate the exit rate from data as $\rho = 0.061$. For the number of products, the observation error is given by 10 percent of one standard deviation of the number of products in the data, i.e., 0.074/10. Similarly, the observation error for the final sales of the new products is 0.12/10. Observation errors are assumed to be i.i.d. normally distributed with zero mean. For the mean of the standard deviation of price observation errors, we assume 50 percent of the standard deviation of the average price, i.e., 0.095/2. Other settings for the estimation remain the same as in Table 2.

Table 8 shows moments for the posterior distributions. The estimation results suggest the existence of search frictions among products by the top two companies. The mean of the matching curvature parameter γ is 1.235. We can calculate the steady state matching probability for an unmatched product as 0.823.[11] About 82 percent of unmatched

[11] Using the estimated parameters in Table 7, we can calculate all the steady state values such as

products can find a retailer.

As shown in Table 8, the bargaining power parameter for the product is 0.765. The optimal price change probability after product entry $1-\alpha$ is 0.023. It implies that only 2.3 percent of products change prices each month after the first price. This number is slightly bigger than the one from Table 3. However, our result suggests that price changes are mainly brought by product entry since companies can set a new price with a probability of 6.1 percent.

As shown in Table 9, for small companies, the demand shock explains 77.53 percent of price variations and it is the largest contribution among the four types of shocks. This is a clear difference from the top two companies as shown in Table 6. It implies that it is difficult to add production costs to their prices for small companies with a small sales share. We reconfirm it in the data. The average price of 147.76 yen is lower than the average new price of 153.74 yen for small companies. These small companies can not increase prices after product entries to production costs and probably discount prices for older products.

7 Concluding Remark

We estimate a model including product entry and exit with search frictions and show that endogenous product entry inducing extensive margins and search frictions are quantitatively important in explaining price variations.

These fundamental elements in explaining price variations are ignored in the official data and conventional models in which product entry and exit are excluded. This disadvantage can misidentify price variations in models and the price index.

$\kappa = 5.21$, $q = 0.287$, $s = 0.823$, $N = 0.825$, $m = 0.05$, $u = 0.175$, $v = 0.06$, and $\theta = 0.394$.

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Table 1: Basic Statistics

	Average	Standard deviation
Number of products	572.92	61.65
Entry rate	0.086	0.025
Exit rate	0.084	0.021
Average price	122.26	5.75
Average new price	118.88	23.37

Note: Monthly base.

Table 2: Prior Distributions

Parameters	Description	Mean	S.D.	Distribution
γ	Matching friction parameter	2	2	Inv. Gamma
b	Bargaining power of producers	0.5	0.1	Beta
α	Probability of price change after entry	0.8	0.1	Beta
ρ_x	Persistence of cost-push shock	0.5	0.15	Beta
ρ_{zb}	Persistence of demand shock	0.5	0.15	Beta
ρ_e	Persistence of matching efficiency shock	0.5	0.15	Beta
ρ_{fe}	Persistence of free entry shock	0.5	0.15	Beta
σ_x	S.D. of cost-push shock	0.2	0.2	Inv. Gamma
σ_{zb}	S.D. of demand shock	0.2	0.2	Inv. Gamma
σ_e	S.D. of matching efficiency shock	0.4	0.4	Inv. Gamma
σ_{fe}	S.D. of free entry shock	0.4	0.4	Inv. Gamma
σ_{err}	S.D. of price observation error	0.038	0.038	Inv. Gamma

Note: S.D. denotes a standard deviation.

Table 3: Posterior Distributions

Parameters	Mean	90 percent interval
γ	0.855	[0.563, 1.108]
b	0.747	[0.706, 0.793]
α	0.986	[0.974, 0.997]
ρ_x	0.473	[0.248, 0.695]
ρ_{zb}	0.176	[0.088, 0.263]
ρ_e	0.299	[0.166, 0.438]
ρ_{fe}	0.352	[0.17, 0.514]
σ_x	0.179	[0.064, 0.301]
σ_{zb}	0.352	[0.322, 0.382]
σ_e	0.159	[0.105, 0.214]
σ_{fe}	0.186	[0.112, 0.259]
σ_{err}	0.039	[0.036, 0.043]
MDD	615.362	

Note: MDD denotes marginal data density.

Table 4: Variance Decomposition in Percent

Variable	Cost-push	Demand	Matching efficiency	Free entry
P_t	2.92	71.71	15.7	9.67
$\tilde{P}_t$	1.74	77.4	13.42	7.44
N_t	0.21	0.3	65.24	34.26
M_t	0.15	0.38	67.48	31.99
θ_t	0.31	0.63	39.81	59.25

Note: Cost-push, Demand, Matching efficiency, and Free entry denote cost-push shock, demand shock, matching efficiency shock, and free entry shock, respectively.

Table 5: Standard Deviation of Prices

$\tilde{P}_t$	S.D.	Change
Exo. model + demand/supply shocks	0.0376	
Baseline model + all shocks	0.0571	51.9
P_t	S.D.	Change
Exo. model + demand/supply shocks	0.0099	
Baseline model + all shocks	0.0149	50.5

Note: Change is evaluated by percent increase from the exogenous entry model. Exo. model denotes an exogenous entry model. S.D. denotes a standard deviation.

Table 6: Posterior Distributions for Top Two Companies

Parameters	Mean	90 percent interval
γ	0.745	[0.516, 0.943]
b	0.711	[0.656, 0.762]
α	0.978	[0.959, 0.997]
ρ_x	0.583	[0.276, 0.838]
ρ_{zb}	0.168	[0.086, 0.236]
ρ_e	0.294	[0.14, 0.467]
ρ_{fe}	0.353	[0.153, 0.551]
σ_x	0.982	[0.075, 2.987]
σ_{zb}	0.414	[0.38, 0.451]
σ_e	0.182	[0.118, 0.243]
σ_{fe}	0.22	[0.126, 0.313]
σ_{err}	0.045	[0.039, 0.051]
MDD	492.269	

Note: MDD denotes marginal data density.

Table 7: Variance Decomposition in Percent for Top Two Companies

Variable	Cost-push	Demand	Matching efficiency	Free entry
P_t	69.63	23.16	4.30	2.91
$\tilde{P}_t$	51.09	39.59	5.82	3.51
N_t	11.68	0.25	56.02	32.05
M_t	6.72	0.35	61.55	31.38
θ_t	14.07	0.52	32.93	52.48

Note: Cost-push, Demand, Matching efficiency, and Free entry denote cost-push shock, demand shock, matching efficiency shock, and free entry shock, respectively.

Table 8: Posterior Distributions for Small Companies

Parameters	Mean	90 percent interval
γ	1.235	[0.753, 1.675]
b	0.765	[0.706, 0.823]
α	0.977	[0.957, 0.995]
ρ_x	0.461	[0.222, 0.681]
ρ_{zb}	0.168	[0.086, 0.236]
ρ_e	0.204	[0.104, 0.294]
ρ_{fe}	0.339	[0.15, 0.493]
σ_x	0.188	[0.064, 0.339]
σ_{zb}	0.792	[0.732, 0.855]
σ_e	0.161	[0.102, 0.218]
σ_{fe}	0.179	[0.099, 0.252]
σ_{err}	0.097	[0.088, 0.105]
MDD	225.364	

Note: MDD denotes marginal data density.

Table 9: Variance Decomposition in Percent for Small Companies

Variable	Cost-push	Demand	Matching efficiency	Free entry
P_t	0.67	77.53	11.72	10.08
$\tilde{P}_t$	0.41	81.45	10.00	8.14
N_t	0.17	1.83	54.67	43.34
M_t	0.12	2.29	55.75	41.84
θ_t	0.18	2.79	41.99	55.04

Note: Cost-push, Demand, Matching efficiency, and Free entry denote cost-push shock, demand shock, matching efficiency shock, and free entry shock, respectively.

Appendix

We provide more details about the Nikkei data and derivations of our models in the Appendix.

A Details of Nikkei Data

A.1 Product Identification

A barcode including the Japanese Article Number (JAN) code is printed on all products and products are distinguished by fairly detailed classifications in Nikkei POS scanner data. In the JAN code, the first seven digits indicate the company code and the last six digits indicate the individual product. When JAN codes are different for the same type of products by the same company, these products are different in terms of packaging, ingredients, etc. In addition, the barcodes provide information about the product category (such as butter, yogurt, or shampoo) and the producer of each product.

A.2 Price

The Nikkei data contains the sales values and quantities sold for each product in each shop on a daily basis. By dividing the sales values by the quantities sold, we calculate the daily price for each product. Based on these individual prices, we calculate an average price for all products and an average price for new products. We use sales values as weights to calculate average prices.

To calculate the average prices, we use price levels. The first reason is that this is the average price that Japanese consumers face in shops to decide on purchases. The second reason is that price dispersion is not large because prices in the Nikkei data are for products in supermarkets where food products and daily necessities are sold and we restrict samples to snack foods.

In details, an average price is calculated as follows. Let $p_{i,s,d}$ and $q_{i,s,d}$ denote the price and the quantity sold of product i at shop s on day d. Then, we compute the price in month t as: $p_{i,s,t} = \frac{\sum_{d \in t} p_{i,s,d} q_{i,s,d}}{\sum_{d \in t} q_{i,s,d}}$. By aggregating this price across products and shops, we calculate an average price $AveragePrice_t$.

$$Average\ Price_t = \frac{\sum_{i \in I, s \in S} price_{ist} weight_{ist}}{\sum_{i \in I, s \in S} weight_{ist}},$$

where $weight_{ist} = \sum_{d \in t} p_{i,s,d} q_{i,s,d}$ and I and S denote the set of products and shops, respectively. We have similar equations for an average price for new products.

A.3 Entry Rate and Exit Rate

In calculating the entry (exit) rate, we define a new (discontinued) product as one for which a transaction is firstly (finally) recorded in a given period. Then, we obtain the number of new (discontinued) products in a given period, which is divided by the total number of products in a given period to calculate entry (exit) rates. Note that these rates are not weighted by sales. We interpret that a new product enters into the market when we observe the new product in at least one shop. We interpret that an existing product exits from the market when no shops sell the existing product. Thus, entry (exit) rates are at the product level. Equations for the entry (exit) rate are given by

$$Entry\ Rate_t = \frac{Number\ of\ New\ Products\ at\ Time\ t}{Total\ Number\ of\ Products\ at\ Time\ t},$$

$$Exit\ Rate_t = \frac{Number\ of\ Discontinued\ Products\ at\ Time\ t}{Total\ Number\ of\ Products\ at\ Time\ t}.$$

A.4 Matching Ratio

The matching ratio calculates the fraction of shops that sell a product. Specifically, the matching ratio is obtained as follows. The Nikkei POS data includes observations at the product-shop level on a daily basis. Using this, we count the number of shops that sell product i at least one day in month t, denoted by $\#\ of\ shops_{i,t}$. Given that product i is classified into category c, we can calculate the maximum number of these numbers across products within category c, denoted by $Max\ \#\ of\ shops_{c,t}$. Then we define the matching ratio of product i in quarter t as:

$$\frac{\#\ of\ shops_{i,t}}{Max\ \#\ of\ shops_{c,t}}.$$

In this paper, we use 25 supermarkets in Tokyo metropolitan area as a denominator. Finally, we take the simple average (or sales-weighted average) across products of these ratios to obtain the time series.

B Steady State

Steady state values for eight endogenous variables given by J, Q, u, v, N, M, q, and θ, are given by following eight equations under given parameters β, ρ, α, b, and γ.

$$J\left[1-\beta(1-\rho)\right] = -X + Z\tilde{P},$$

$$Q\left[1-\beta(1-\rho-q)\right] = Z^B - Z\tilde{P},$$

$$bQ = (1-b)J,$$

$$u = 1 - N,$$

$$\rho N = M,$$

$$M = \frac{uv}{(u^\gamma + v^\gamma)^{\frac{1}{\gamma}}},$$

$$\theta = \frac{v}{u},$$

$$q = \frac{M}{u},$$

where $Q = V^1 - V^0$ and X, Z, $\tilde{P}$, and Z^B are exogenously given.

C Complete Set of Model and Linearization

C.1 Price Aggregator

Price aggregation is given by

$$N_t P_t = \alpha(1-\rho)N_{t-1}P_{t-1} + (1-\alpha)(1-\rho)N_{t-1}\tilde{P}_t + M_{t-1}\tilde{P}_t, \tag{14}$$

where α is a probability of no optimal price change at time t.

We assume the following matching function.

$$M_t = \chi_t \frac{u_t v_t}{(u_t^\gamma + v_t^\gamma)^{\frac{1}{\gamma}}}, \tag{15}$$

where γ is a positive parameter and χ_t is a matching efficiency shock, where $\chi = 1$ in a steady state.

In a steady state, we have

$$NP = \alpha(1-\rho)NP + (1-\alpha)(1-\rho)N\tilde{P} + M\tilde{P}.$$

We also have an equation for the number of matches.

$$N_{t+1} = (1-\rho)N_t + M_t. \tag{16}$$

In a steady state,

$$\rho N = M.$$

Thus, we have

$$NP = \alpha(1-\rho)NP + (1-\alpha)(1-\rho)N\tilde{P} + \rho N\tilde{P},$$
$$\frac{\tilde{P}}{P} = 1.$$

By linearizing Eq. (14), we have

$$\begin{aligned}\hat{N}_t + \hat{P}_t &= \alpha(1-\rho)\left(\hat{N}_{t-1} + \hat{P}_{t-1}\right) \\ &\quad +(1-\alpha)(1-\rho)\left(\hat{N}_{t-1} + \hat{\tilde{P}}_t\right) \\ &\quad +\rho\left(\hat{M}_{t-1} + \hat{\tilde{P}}_t\right).\end{aligned}$$

where $\hat{M}_t$ denotes a new match as

$$\hat{M}_t = \frac{1}{1+\theta^\gamma}\hat{\theta}_t + \hat{u}_t + \hat{\chi}_t, \tag{17}$$

and we use a definition of a market tightness and

$$u_t = 1 - N_t,$$

and

$$u\hat{u}_t = -N\hat{N}_t. \tag{18}$$

By linearizing Eq. (16), we have

$$\begin{aligned} N\hat{N}_t &= (1-\rho)N\hat{N}_{t-1} + M\hat{M}_{t-1}, \\ \hat{N}_t &= (1-\rho)\hat{N}_{t-1} + \rho\hat{M}_{t-1}. \end{aligned} \tag{19}$$

Thus, we have

$$[1-\alpha(1-\rho)]\,\hat{\tilde{P}}_t = \hat{P}_t - \alpha(1-\rho)\hat{P}_{t-1}$$

C.2 Value Functions

From a free entry condition given by

$$\kappa_t = \beta s_t \mathrm{E}_t J_{t+1}(\tilde{P}_{t+1}),$$

where κ_t is a free entry shock and it is κ in a steady state, we have

$$\hat{\kappa}_t = \hat{s}_t + \mathrm{E}_t\hat{J}_{t+1}(\tilde{P}_{t+1}). \tag{20}$$

Together with a definition of a market tightness, we have

$$s_t = \frac{M_t}{v_t} = \frac{\chi_t}{\left[1+\left(\frac{v_t}{u_t}\right)^\gamma\right]^{\frac{1}{\gamma}}} = \frac{\chi_t}{[1+\theta_t^\gamma]^{\frac{1}{\gamma}}},$$

then we have

$$\hat{s}_t = -\frac{\theta^\gamma}{1+\theta^\gamma}\hat{\theta}_t + \hat{\chi}_t,$$

and so

$$\hat{J}_{t+1}(\tilde{P}_{t+1}) = \frac{\theta^\gamma}{1+\theta^\gamma}\hat{\theta}_t + \hat{\kappa}_t - \hat{\chi}_t. \tag{21}$$

For a product with a contract $\tilde{P}_t$ at time t,

$$J_t(\tilde{P}_t) = Z\tilde{P}_t - X_t + \beta(1-\rho)\mathrm{E}_t\left[\alpha J_{t+1}(g\tilde{P}_t) + (1-\alpha)J_{t+1}(\tilde{P}_{t+1})\right].$$

Iterating forward, we have

$$\begin{aligned} J_t(\tilde{P}_t) \;\; = \;\; & -X_t + \alpha\beta(1-\rho)\mathbb{E}_t X_{t+1} + \alpha^2\beta^2(1-\rho)^2\mathbb{E}_t X_{t+2} + \cdots \\ & -Z\tilde{P}_t - \alpha\beta(1-\rho)Z\tilde{P}_t - \alpha^2\beta^2(1-\rho)^2 Z\tilde{P}_t - \cdots \\ & +\beta(1-\rho)(1-\alpha)\mathbb{E}_t J_{t+1}(\tilde{P}_{t+1}) + \beta^2(1-\rho)^2\alpha(1-\alpha)\mathbb{E}_t J_{t+2}(\tilde{P}_{t+2}) \\ & +\beta^3(1-\rho)^3\alpha^2(1-\alpha)\mathbb{E}_t J_{t+3}(\tilde{P}_{t+3}) + \cdots . \end{aligned}$$

We also have a similar equation for $J_{t+1}(\tilde{P}_{t+1})$ and then

$$\begin{aligned} & J_t(\tilde{P}_t) - \beta(1-\rho)J_{t+1}(\tilde{P}_{t+1}) \\ = \;\; & -X_t + \frac{Z}{1-\beta(1-\rho)\alpha}\tilde{P}_t - \frac{Z\beta(1-\rho)\alpha}{1-\beta(1-\rho)\alpha}\mathbb{E}_t\tilde{P}_{t+1}. \end{aligned}$$

By linearizing it, we have

$$\begin{aligned} & J\hat{J}_t(\tilde{P}_t) - J\beta(1-\rho)\mathbb{E}_t\hat{J}_{t+1}(\tilde{P}_{t+1}) \\ = \;\; & -X\hat{X}_t + \frac{Z\tilde{P}}{1-\beta(1-\rho)\alpha}\hat{\tilde{P}}_t - \frac{Z\tilde{P}\beta(1-\rho)\alpha}{1-\beta(1-\rho)\alpha}\mathbb{E}_t\hat{\tilde{P}}_{t+1}. \end{aligned} \tag{22}$$

The value function for a retailer that is matched with a product and has a contract $\tilde{P}_t$ at time t

$$V_t^1(\tilde{P}_t) = Z_t^B - Z\tilde{P}_t + \beta\mathrm{E}_t\left[\alpha(1-\rho)V_{t+1}^1(\tilde{P}_t) + (1-\alpha)(1-\rho)V_{t+1}^1(\tilde{P}_{t+1}) + \rho V_{t+1}^0\right].$$

where V^0 is the value function for a retailer without a match. Let q_t be the probability that the retailer finds a producer. The value function for a retailer without a match is

$$V_t^0 = \beta\mathrm{E}_t\left[q_t V_{t+1}^1(\tilde{P}_{t+1}) + (1-q_t)V_{t+1}^0\right].$$

Iterating $V_t^1(\tilde{P}_t) - V_t^0$ forward, we have

$$V_t^1(\tilde{P}_t) - V_t^0 = Q_t(\tilde{P}_t) = Z_t^B - \frac{Z}{1-\beta(1-\rho)\alpha}\tilde{P}_t + \frac{\alpha\beta(1-\rho)Z}{1-\beta(1-\rho)\alpha}\mathrm{E}_t\tilde{P}_{t+1} + \beta(1-\rho-q_t)\mathrm{E}_t Q_{t+1}(\tilde{P}_{t+1}),$$

$$Q_t(\tilde{P}_t) - \beta(1-\rho)\mathrm{E}_t Q_{t+1}(\tilde{P}_{t+1}) = Z_t^B - \frac{Z}{1-\beta(1-\rho)\alpha}\tilde{P}_t + \frac{\alpha\beta(1-\rho)Z}{1-\beta(1-\rho)\alpha}\mathrm{E}_t\tilde{P}_{t+1} - q_t\beta\mathrm{E}_t Q_{t+1}(\tilde{P}_{t+1}),$$

By linearizing it, we have

$$\begin{aligned} & Q\hat{Q}_t(\tilde{P}_t) - Q\beta(1-\rho)\mathrm{E}_t\hat{Q}_{t+1}(\tilde{P}_{t+1}) \\ = \ & Z^B\hat{Z}_t^B - \frac{Z\tilde{P}}{1-\beta(1-\rho)\alpha}\hat{\tilde{P}}_t + \frac{Z\tilde{P}\alpha\beta(1-\rho)}{1-\beta(1-\rho)\alpha}\mathrm{E}_t\hat{\tilde{P}}_{t+1} - \beta Qq\left[\mathrm{E}_t\hat{Q}_{t+1}(\tilde{P}_{t+1}) + \hat{q}_t\right], \end{aligned}$$

$$\begin{aligned} & Q\hat{Q}_t(\tilde{P}_t) - Q\beta(1-\rho)\mathrm{E}_t\hat{Q}_{t+1}(\tilde{P}_{t+1}) \\ = \ & Z^B\hat{Z}_t^B - \frac{Z\tilde{P}}{1-\beta(1-\rho)\alpha}\hat{\tilde{P}}_t + \frac{Z\tilde{P}\alpha\beta(1-\rho)}{1-\beta(1-\rho)\alpha}\mathrm{E}_t\hat{\tilde{P}}_{t+1} - \beta Qq\left(\hat{\theta}_t + \hat{\kappa}_t\right), \end{aligned} \tag{23}$$

where we use a free entry condition and

$$\hat{Q}_t(\tilde{P}_t) = \hat{J}_t(\tilde{P}_t), \tag{24}$$

$$\hat{q}_t - \hat{s}_t = \hat{\theta}_t.$$

Then, by subtracting Eq. (22) multiplied by $1-b$ from Eq. (23) multiplied by b, we have

$$\begin{aligned} & \frac{Z\tilde{P}}{1-\beta(1-\rho)\alpha}\hat{\tilde{P}}_t \\ = \ & \frac{Z\tilde{P}\alpha\beta(1-\rho)}{1-\beta(1-\rho)\alpha}\mathrm{E}_t\hat{\tilde{P}}_{t+1} - b\beta qQ\left(\hat{\theta}_t + \hat{\kappa}_t\right) + (1-b)X\hat{X}_t + bZ^B\hat{Z}_t^B, \end{aligned} \tag{25}$$

since a free entry condition and

$$bQ = (1-b)J,$$

due to a nash bargaining. Form Eq. (20) and Eq. (25), we can derive Phillips curve.

Also, by summing up Eq. (22) and Eq. (23), we finally have

$$
\begin{aligned}
& J\mathrm{E}_t\hat{J}_{t+1}(\tilde{P}_{t+1}) \\
= \;& \beta(1-\rho)J\mathrm{E}_t\hat{J}_{t+2}(\tilde{P}_{t+2}) - b\beta Qq\left(\mathrm{E}_t\hat{\theta}_{t+1} + \mathrm{E}_t\hat{\kappa}_{t+1}\right) + bZ^B\mathrm{E}_t\hat{Z}^B_{t+1} - bX\mathrm{E}_t\hat{X}_{t+1},
\end{aligned}
$$

$$
\begin{aligned}
\hat{\theta}_t \;=\;& \beta\left[1-\rho-(1-b)q\frac{1+\theta^\gamma}{\theta^\gamma}\right]\mathrm{E}_t\hat{\theta}_{t+1} \\
+\;& \beta\frac{1+\theta^\gamma}{\theta^\gamma}\left[1-\rho-(1-b)q\right]\mathrm{E}_t\hat{\kappa}_{t+1} - \frac{1+\theta^\gamma}{\theta^\gamma}\hat{\kappa}_t \\
-\;& \beta(1-\rho)\frac{1+\theta^\gamma}{\theta^\gamma}\mathrm{E}_t\hat{\chi}_{t+1} + \frac{1+\theta^\gamma}{\theta^\gamma}\hat{\chi}_t + \frac{bZ^B}{J}\frac{1+\theta^\gamma}{\theta^\gamma}\mathrm{E}_t\hat{Z}^B_{t+1} - \frac{bX}{J}\frac{1+\theta^\gamma}{\theta^\gamma}\mathrm{E}_t\hat{X}_{t+1},
\end{aligned}
\tag{26}
$$

where we use $bQ = (1-b)J$ and Eq. (21).

C.3 Closed System of Economy

A closed economy consists of six equations after linearization, Eq. (17), Eq. (18), Eq. (19), Eq. (20), Eq. (25), and Eq. (26) for six variables, $\hat{\tilde{P}}_t$, $\hat{P}_t$, $\hat{\theta}_t$, $\hat{N}_t$, $\hat{M}_t$, and $\hat{u}_t$.

In an estimation for parameters, we use these six equations including parameters and steady state values. Steady state values are simultaneously determined by estimation by estimated parameters.