Optimal Monetary Policy in a Liquidity Trap:

Evaluations for Japan's Monetary Policy

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Abstract

This paper shows that the Bank of Japan (BOJ)'s monetary policy for the COVID-19 pandemic period, "Inflation-Overshooting Commitment," shares several similarities with optimal monetary policy in a liquidity trap.

We calibrate a hybrid new Keynesian model that includes inflation persistence by Japanese parameters and compare the BOJ's monetary policy with optimal monetary policy under commitment. Optimal monetary policy prolongs the zero interest rate until the second quarter of 2024 even after inflation rates sufficiently exceed 2 percent. Similarly, the BOJ continues the zero interest rate policy at least until the second quarter of 2024. We show that recent high inflation rates can be explained by a prolonged zero interest rate policy under the commitment policy. The current average inflation rates from 2023Q2 to 2024Q1 reach 2.5 percent and 2.4 percent in the data and the model with optimal monetary policy, respectively.

Our conclusion mostly holds for a variety of situations with Japanese parameters, such as a low output gap response to the real interest rate, discounted IS curve, alternative inflation persistences, interest-rate smoothing for the policy rate, an alternative specification of the Phillips curve, inflation target, and low/high anchored inflation expectations and natural interest rates.

JEL Classification: E31; E52; E58; E61
Keywords: liquidity trap; optimal monetary policy; inflation persistence; commitment policy; forward guidance

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1 Introduction

In September 2016, the Bank of Japan (BOJ) introduces the "Inflation-Overshooting Commitment" under the zero interest rate policy. In the inflation-overshooting commitment, the BOJ commits to continuing monetary easing until the year-on-year CPI inflation rate stably exceeds the 2 percent target. As discussed below, this commitment policy works as an optimal monetary policy to raise the inflation rate and its expectations, and to lower the real interest rate. Due to the Covid-19 pandemic in early 2020, the economy temporarily declined, but high inflation has subsequently emerged after it in Japan. Now, the BOJ faces an exit policy from a liquidity trap under the commitment, and we evaluate whether the BOJ conducts optimal monetary policy.

In this paper, we introduce optimal monetary policy under commitment in a liquidity trap for a hybrid new Keynesian model including inflation persistence. Then, we apply our model to Japan's monetary policy and show that optimal monetary policy replicates the BOJ's monetary policy and the Japanese economy during and after the pandemic period. The BOJ's inflation-overshooting commitment policy has several similarities with optimal monetary policy in a liquidity trap. Under both policies, the zero interest rate policy continues even after inflation rates sufficiently exceed 2 percent. Optimal monetary policy keeps the zero interest rate until the second quarter of 2024. The BOJ continues the zero interest rate policy at least until the second quarter of 2024 under the commitment. Recent high inflation rates can be explained by a prolonged zero interest rate policy. The current average inflation rates from 2023Q2 to 2024Q1 reach 2.5 percent and 2.4 percent in the data and the model, respectively. A simple interest rate rule without commitment can not make such high inflation rates.

Our paper is not the first paper on optimal monetary policy in a liquidity trap. Eggertsson and Woodford (2003b,a) and Jung et al. (2001, 2005) show that the optimal commitment policy is history dependent and a central bank continues a zero interest rate policy even after the natural interest rate turns positive. Adam and Billi (2006, 2007) and Nakov (2008) introduce stochastic shocks into the zero interest rate policy analyses under the optimal commitment policy as well as the discretionary policy. Nakajima

(2008) extends Eggertsson and Woodford (2003b) to a two-country model and analyzes optimal monetary policy under co-ordination in a liquidity trap.[1] Werning (2011) shows that the future consumption boom as well as the future high inflation play important roles in mitigating a liquidity trap. Evans et al. (2015) show an exit strategy from a liquidity trap under optimal discretionary policy by using a purely forward-looking model and a purely backward-looking model. As an independent work for a deterministic shock, Michau (2019) shows optimal monetary and fiscal policy in a liquidity trap for a new Keynesian Phillips curve including a lagged inflation rate. He shows that a central bank terminates the zero interest rate policy earlier under a higher inflation persistence.[2] All these papers are the foundation for our paper and our contribution is to show optimal monetary policy in Japan after the last pandemic in various situations.

Our paper is related to the literature on the Japanese economy and monetary policy for the post-Covid pandemic periods. Ikeda et al. (2022) analyze the inflation behavior before and after the pandemic. They argue that cost-push pressures, such as commodity price hikes and yen depreciation, temporarily raise the inflation rate after the pandemic. Its effect, however, would disappear in a short period. Rather than the shocks, they discuss that the medium- to long-term inflation expectations can be a key factor for inflation rates. They suggest that monetary policy can contribute to inflation dynamics after the pandemic. Nakamura et al. (2024) apply Blanchard and Bernanke's (2024) model to Japan.[3] They argue that Japan's high post-pandemic inflation is mainly due to product market-specific shocks rather than economic activities such as labor market

[1] Additionally, Hasui et al. (2019) is another study on the optimal monetary policy in a liquidity trap for the Japanese economy. They analyze the optimal commitment policy in a liquidity trap taking into account the adaptive expectations. Our paper differs from these studies by incorporating inflation persistence.

[2] Our companion paper, Hasui et al. (2024), shows that the Fed's exit policy from a liquidity trap is optimal monetary policy by using a new Keynesian model with inflation persistence.

[3] Blanchard and Bernanke (2024) construct an empirical model including wages and labor market tightness. Their findings indicate that the factors driving post-pandemic inflation are similar across different countries.

tightness.[4] Kishaba and Okuda (2023) estimate the slope of the Phillips curve in Japan using the approach of Hazell et al. (2022). They find that the slope for Japan remains flat during the post-pandemic period. It implies that the monetary policy can be still less effective in Japan after the pandemic and need an aggressive policy such as the commitment policy to raise inflation rates.

There are analyses on monetary policy for pre-pandemic periods in Japan. Michelis and Iacoviello (2016) interpret the BOJ's 2 percent inflation target as an aggressive monetary easing policy. They simulate a DSGE model calibrated by the results of their empirical model and show that while raising the inflation target is effective in escaping from deflation, an inflation rate shows a sluggish response toward the target level. In particular, Kawamoto et al. (2024) analyze the BOJ's inflation-overshooting commitment as an implementation of the "makeup strategy" using an estimated model including a hybrid Phillip curve, backward-looking IS curve, and variants of Taylor-type rules. They show that a prolonged low interest rate policy with inflation overshooting can work as the makeup strategy from the perspective of early achievement of the inflation target.

This study focuses on inflation persistence in the Phillips curve and optimal monetary policy for Japan. For inflation persistence in Japan, estimations vary according to different studies based on DSGE models and econometric models with different sample periods as shown in Sugo and Ueda (2008); Hirose and Kurozumi (2012); Ichiue et al. (2013); Kaihatsu and Kurozumi (2014); Michelis and Iacoviello (2016); Hirose (2020); Kawamoto et al. (2023); Kawamoto et al. (2024). For example, the recent BOJ's papers of Kawamoto et al. (2023) and Kawamoto et al. (2024) set high inflation persistence in models and evaluate the "Quantitative and Qualitative Monetary Easing" and the "Inflation-Overshooting Commitment." On the other hand, Hirose (2020) and Hirose and Kurozumi (2012) show low inflation persistence. Moreover, Nakamura et al. (2024), in their estimation of the Blanchard and Bernanke (2024) model for Japan, reveal that inflation expectations are largely explained by its lagged variables and the sum of coefficients on the lagged variables is 0.981 for short-term and 0.994 for long-term, respectively.

[4] They argue that this result mirrors the findings of Blanchard and Bernanke (2024) for the US.

This implies that inflation expectations are highly persistent in Japan compared to the US and the sluggish inflation expectation contributes to high inflation persistence. In the paper, we investigate several cases of inflation persistence.

For optimal monetary policy, Kawamoto et al. (2024) summarize that BOJ's "Quantitative and Qualitative Monetary Easing with Yield Curve Control" implemented in the post-Covid pandemic periods consists of the "Yield Curve Control (YCC)" and the "Inflation-Overshooting Commitment." They analyze the effectiveness of the inflation-overshooting commitment by continuing the zero interest rate policy in models. As in Kawamoto et al. (2024), we mainly focus on the inflation-overshooting commitment with the prolonged zero interest rate policy. A clear difference from Kawamoto et al. (2024) is that we introduce optimal monetary policy in a liquidity trap and they assume the Taylor-type rules. Our analysis naturally covers the YCC policy since expectations for future monetary policy fundamentally determine the yield curve.

Given these backgrounds, analyzing optimal commitment policy in economic models with high inflation persistence is suitable for the Japanese economy. As shown in Section 4.2, we assume high inflation persistence to replicate the recent inflation rates in Japan. In addition, in Section 5, we present comprehensive analyses of monetary policy in Japan. For example, we show the cases of low intertemporal elasticity of substitution and discounted IS curve. These cases show a better fit of the model to the output gap data. Moreover, we show alternative cases for inflation persistence. The results show that the higher the inflation persistence, the closer the inflation dynamics in the model with the actual data. We also show the cases of an alternative specification for the Phillips curve, a simple monetary policy rule without commitment, a welfare loss function including interest-rate smoothing, inflation target, and alternative scenarios for natural interest rate and anchored inflation expectation.

The remainder of the paper proceeds as follows. Section 2 presents a model with inflation persistence. Section 3 derives optimal monetary policy in a liquidity trap. In Section 4, we show optimal monetary policy for Japan for the pandemic. Section 5 shows comprehensive analyses of monetary policy in Japan. Section 6 concludes.

2 The Model

We use a new Keynesian model following Woodford (2003a) and Eggertsson and Woodford (2006) and omit detailed explanations for the model. The macroeconomic structure is expressed by the two equations:

$$x_t = \mathrm{E}_t x_{t+1} - \chi \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n \right), \tag{1}$$

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \beta \left(\mathrm{E}_t \pi_{t+1} - \gamma \pi_t \right) + \mu_t, \tag{2}$$

where χ is the intertemporal elasticity of substitution of expenditure, β is a discount factor, γ $(0 \leq \gamma \leq 1)$ is the degree of inflation persistence, and

$$\kappa = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \frac{\omega + \chi^{-1}}{1+\omega\theta},$$

where ω is the elasticity of firm's real marginal cost and θ is an elasticity of substitution across goods. It should be noted that a slope of the Phillips curve κ depends on price stickiness α. x_t, i_t and π_t denote the output gap, the nominal interest rate (or policy rate), and the rate of inflation in period t, respectively. The expectations operator E_t covers information available in period t. r_t^n is the natural rate of interest and works as the shock. μ_t is the cost-push shock.

Equation (1) is the forward-looking IS curve as shown in Clarida et al. (1999) and Woodford (2003a). The IS curve states that the current output gap is determined by the expected value of the output gap and the deviation of the current real interest rate, defined as $i_t - \mathrm{E}_t \pi_{t+1}$, from the natural interest rate.

Equation (2) is the hybrid Phillips curve. When $\gamma = 0$, the hybrid Phillips curve turns into a purely forward-looking Phillips curve, where current inflation is dependent on expected inflation and the current output gap. When $0 < \gamma \leq 1$, the Phillips curve is both forward-looking and backward-looking, and the current inflation rate depends on the lagged inflation rate, as well as the expected inflation and the current output gap. When γ is closer to 1, the coefficient on the lagged inflation rate is closer to 0.5. Following the indexation rule in Woodford (2003a), some firms that cannot reoptimize their own goods prices adjust current prices based on the past inflation rate.

Next, we show the central bank's intertemporal optimization problem. The central bank sets the nominal interest rate i_t so as to minimize the approximated welfare loss $\mathcal{L}_t$ defined as

$$\mathcal{L}_t = \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \mathrm{L}_{t+j}, \tag{3}$$

where L_t is the period loss function obtained by second-order approximation of the household utility function. In an economy with inflation inertia, Woodford (2003a) shows that L_t is given by

$$\mathrm{L}_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda_x x_t^2, \tag{4}$$

where $\lambda_x = \frac{\kappa}{\theta}$ is a weight for the output gap and a non-negative parameter. A central bank needs to stabilize $\pi_t - \gamma\pi_{t-1}$ in approximation rather than the inflation rate itself from the target rate when inflation exhibits persistence. In an economy with indexation on inflation rates, price dispersion comes from an environment where some firms not reoptimizing their prices follow the past inflation rate with a certain degree γ and other firms reoptimize prices. Therefore, to minimize price dispersion, a central bank needs to set the current inflation rate so as to be close to the lagged inflation rate with adjustment by γ. This is eventually consistent with the BOJ's inflation-overshooting commitment to allow inflation rates to flexibly exceed a target level of inflation rate. However, it notes that we show optimal monetary policy that maximizes the household's utility regardless of the approximation.

Finally, we give a nonnegativity constraint on the nominal interest rate:

$$i_t \geq 0. \tag{5}$$

The central bank maximizes equation (3) subject to equations (1), (2), and (5).

3 Optimal Monetary Policy in a Liquidity Trap

We follow Michau (2019) and Hasui et al. (2024) and analytically show optimal monetary policy under commitment in a liquidity trap and clarify the optimal exit strategy.[5] The

[5]Michau (2019) assumes deterministic shocks and Hasui et al. (2024) introduce stochastic shocks.

optimization problem is represented by the following Lagrangian form:

$$\mathcal{L} = \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \begin{array}{c} \left(\pi_{t+j} - \gamma \pi_{t+j-1}\right)^2 + \lambda_x x_{t+j}^2 \\ -2\phi_{1t+j} \left[x_{t+j+1} - \chi \left(i_{t+j} - \pi_{t+j+1} - r_{t+j}^n \right) - x_{t+j} \right] \\ -2\phi_{2t+j} \left[\kappa x_{t+j} + \beta \left(\pi_{t+j+1} - \gamma \pi_{t+j} \right) - \pi_{t+j} + \gamma \pi_{t+j-1} \right] \end{array} \right\},$$

where ϕ_1 and ϕ_2 are the Lagrange multipliers associated with the IS constraint and the Phillips curve constraint, respectively. We differentiate the Lagrangian with respect to π_t, x_t, and i_t under the nonnegativity constraint on nominal interest rates to obtain the first-order conditions:

$$-\beta\gamma \left(\mathrm{E}_t \pi_{t+1} - \gamma \pi_t \right) + \pi_t - \gamma \pi_{t-1} - \beta^{-1} \chi \phi_{1t-1} - \beta\gamma \mathrm{E}_t \phi_{2t+1} + \left(\beta\gamma + 1 \right) \phi_{2t} - \phi_{2t-1} = 0, \quad (6)$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1} \phi_{1t-1} - \kappa \phi_{2t} = 0, \quad (7)$$

$$i_t \phi_{1t} = 0, \quad (8)$$

$$\phi_{1t} \geq 0, \quad (9)$$

$$i_t \geq 0. \quad (10)$$

Equations (8), (9), and (10) are conditions for the nonnegativity constraint on nominal interest rates. The above five conditions, together with the IS curve of equation (1) and the hybrid Phillips curve of equation (2), determine the loss minimization. The optimal interest rate is set by these conditions each period. We also need initial conditions for all variables being zero except the nominal interest rate, which takes a positive value in the steady-state. When the nonnegativity constraint is not binding, i.e., $i_t > 0$, the Lagrange multiplier ϕ_{1t} becomes zero by the Kuhn-Tucker condition in equation (8), and the interest rate is determined by the conditions given by equations (1), (2), (6), and (7) with $\phi_{1t} = 0$. When the nonnegativity constraint is binding, i.e., $i_t = 0$, the interest rate is simply set to zero. The interest rate remains zero at least until the Lagrange multiplier ϕ_{1t} becomes zero.

To obtain the path of variables under optimal monetary policy in a liquidity trap, we need numerical simulations because of the nonnegativity constraint on nominal interest rates. Equations (6) and (7) define the qualitative characteristics of optimal monetary policy in a liquidity trap and an economy with persistent inflation.

The first feature is that, due to the central bank's objective to minimize the change in inflation rates $\pi_t - \gamma\pi_{t-1}$, the optimality condition includes terms to smooth inflation rates as shown in equation (6). A strong commitment to inflation smoothing is motivated by both the expected and current changes in inflation rates. In an economy with inflation persistence, less weight is imposed on the deviation of inflation rates from a target level than in an economy without inflation persistence. Thus, agents expect more accommodative stance of the central bank against inflation and a high inflation rate accelerates along with a high expected inflation rate.

The second feature of optimal monetary policy is forward-looking terms associated with introducing inflation persistence into the model. The central bank's monetary policy is determined by the forecast of future inflation rates and the output gap. There are two channels to make optimal monetary policy forward-looking.

The first channel is given by the parameter γ on the future inflation rate in equation (6). Optimal monetary policy in a model with inflation persistence should respond to the expected inflation rate. The second channel is given by the parameter γ in equation (6) on the Lagrange multiplier ϕ_{2t+1} that is related to the future output gap and a future zero interest rate condition. Note that the optimality condition includes the backward-looking variables, which induces history dependent policy as in the standard new Keynesian model. Theoretically, both forward-looking and backward-looking elements determine the optimal path of the nominal interest rates, including the optimal exit from a liquidity trap.

4 Optimal Monetary Policy for Japan

4.1 Naïve Calibration for Japanese Economy

Parameter values are listed in Table 1 as a naïve calibration for optimal monetary policy analysis. We borrow these parameters from representative papers for the Japanese economy. We would like to emphasize that the parameters can change under the recent economic environment in the end of a liquidity trap and the occurrence of the pandemic. We comprehensively answer this point in Section 5.

Sugo and Ueda (2008) estimate a DSGE model for Japanese economy and show that $\alpha = 0.875$, $\omega = 2.149$, and $\theta = 6$.[6] Then, we have $\kappa = 0.0048$, and $\lambda_x = 0.0008$. We set $\chi = \frac{1}{1.548} = 0.646$ following Iiboshi et al. (2022).[7]

For inflation persistence, the recent BOJ's paper Kawamoto et al. (2024) use the BOJ's small-size projection model in which a coefficient on the lagged inflation rate is 0.85 to evaluate the BOJ's inflation-overshooting commitment policy. Moreover, Kawamoto et al. (2023) use the BOJ's macroeconomic model to evaluate the quantitative and qualitative monetary easing policy. They estimate a coefficient on the lagged inflation rate in the Phillips curve as 0.69. These BOJ's papers show high inflation persistence in Japan. Sugo and Ueda (2008) also estimates γ as high as 0.862. As emphasized in Bank of Japan (2024), inflation expectation itself is largely adaptive in Japan and it implies that a coefficient on the lagged inflation rate can be large. These imply that $\gamma = 1$ is still conservative in describing inflation persistence since $\gamma = 1$ implies about 0.5 for a coefficient on the lagged inflation rate as shown in equation (2). For the naïve calibration, we use $\gamma = 1$ and implement the comprehensive analysis for alternative inflation persistences in Section 5.[8] Woodford (2004) shows that the model and all conditions for optimal monetary policy eventually do not change when we set $\gamma = 1$ even for a non-zero inflation target due to $\pi_t - \gamma\pi_{t-1}$ terms in the model.

The long-run nominal interest rate in the model is given by a sum of an anchored inflation expectation and the natural rate of interest. Osada and Nakazawa (2024) estimate

[6] Mukoyama et al. (2021) also estimate high price stickiness as $\alpha = 0.82$.

[7] The estimates for the intertemporal elasticity of substitution in DSGE models vary and Cashin and Unayama (2016) show a low estimate of 0.21. Section 5.1 presents the simulation results for the case where χ is 0.21.

[8] In a new Keynesian model, setting a higher value for inflation persistence is not uncommon. For example, Nakamura and Steinsson (2018) set inflation persistence to the maximum value of 1 in order to replicate the sluggish inflation response in the US economy.

the principal component-based composite index of inflation expectations for different forecast horizons and show that these expectations are about 1.5 percent at the end of 2023. Bank of Japan (2024) shows that the break-even inflation rate, which is the yield spread between fixed-rate coupon-bearing JGBs and inflation-indexed JGBs and captures inflation expectation in financial markets, is about 1.5 percent in April 2024. Thus, we set the anchored inflation expectation at 1.5.

For the natural interest rate in the steady-state, Bank of Japan (2024) shows a variety of estimates because it is difficult to specify an exact natural interest rate. The BOJ shows that the latest estimates of the natural interest rate are distributed around -0.5 in 2023 and we use it for the naïve calibration. Therefore, we set a nominal interest rate at 1.0 percent annually in the steady-state, and a discount factor, i.e., an inverse of the nominal interest rate, is given by $\beta = 0.9975$.

We show alternative cases for the natural interest rate and an anchored inflation expectation in the comprehensive analysis in Section 5.[9]

In simulations, we interpret the second quarter of 2020 as the starting point since we observe the largest negative shocks for the output gap and an inflation rate by the pandemic. The output gap is -6.3 and the inflation rate is -2.8 annually in the second quarter of 2020.[10] The pandemic induces a very large size, but a very short-term shock. We interpret that the BOJ focuses on the exit policy to these large negative shocks. Regarding shocks for the simulation, we give one-time negative natural rate shock and one-time negative cost-push shock without shock persistence as Eggertsson and Woodford

[9]It should be noted that for the naïve calibration, we can set a nominal interest rate at 1.0 percent and decompose it by the anchored inflation expectation at 2, the BOJ's inflation target, and the natural interest rate at -1, the lower bound of estimation. In this case, the simulation result does not change from Figure 1.

[10]We use the Real Gross Domestic Product (Expenditure), Quarterly, Seasonally Adjusted Annual Rate for the output gap. We make a trend series of one-year moving averages and calculate a gap from the trend series to real GDP. For inflation rates, we use the Consumer Price Index for all items, less fresh food, seasonally adjusted. We calculate an annual inflation rate by a growth rate from a previous period. For the BOJ's policy rate, we use the call rate, uncollateralized overnight, average, annually.

(2003b) to match models to the data for an inflation rate and the output gap at the second quarter of 2020, as shown in Figure 1.[11] The simulations are perfect foresight and we use Dynare to run simulations.[12]

4.2 Naïve Analysis under Optimal Monetary Policy for Japan

Figure 1 shows inflation rates, the output gap, and policy rates under optimal monetary policy and these Japanese data from the second quarter of 2020 to the fourth quarter of 2024.[13] The data ends in the first quarter of 2024. We observe that the BOJ's monetary policy shares several common points with optimal monetary policy in a liquidity trap. The zero interest rate policy continues even after inflation rates sufficiently exceed 2 percent. This is consistent with the BOJ's inflation-overshooting commitment that allows inflation rates to stably exceed the 2 percent target. Optimal monetary policy keeps the zero interest rate policy until the second quarter of 2024. The BOJ continues the zero interest rate at least until the second quarter of 2024 under the commitment. Recent high inflation rates can be explained by a prolonged zero interest rate policy. The current

[11] We assume -12.8 percent of the natural rate shock and -0.97 percent of cost-push shock at a time zero as a quarterly base. In simulations, we use the inflation rate data at the first quarter of 2020 to an inflation lag in the model in the period of 0. Before shocks occur, other variables are set to zero. Year-on-year growth rate of the gross domestic product is -9.8 percent in the second quarter of 2020, which is a very rare event and the largest drop after 1985 in the sample period that we can investigate by recent available data. Then, by combining the small parameter for the intertemporal elasticity of substitution, we need such a large negative natural rate shock (demand shock)

[12] We extend a code by Johannes Pfeifer for optimal monetary policy in a liquidity trap, JohannesPfeifer/DSGE_mod/blob/master/Gali_2015/Gali_2015_chapter_5_commitment_ZLB.mod. Our code is available upon your request. On the other hand, Adam and Billi (2006, 2007) and Nakov (2008) conduct simulations using the stochastic method. In the analysis with the stochastic method, compared to the deterministic method, the uncertainty of hitting the zero lower bound amplifies the impact of adverse shocks. Thus, the zero interest rate policy becomes longer under stochastic shocks. Furthermore, the stochastic steady-state may deviate from the deterministic steady-state as shown in Hills et al. (2019).

[13] Inflation rates and policy rates are annual in figures.

average inflation rates from 2023Q2 to 2024Q1 reach 2.5 percent and 2.4 percent in the data and the model, respectively. The simulation also reveals that very quick inflation rises and very high inflation rates such as over 4 percent can not be explained by the model nor optimal monetary policy. These are given by positive cost-push shocks as discussed by Ikeda et al. (2022) and Nakamura et al. (2024).

5 Comprehensive Analysis

5.1 Low Elasticity of Demand to Real Interest Rate

A response of the output gap to the real interest rate is a key parameter for optimal monetary policy in a liquidity trap. In Japan, deflation and low growth continue for a few decades under the zero interest rate policy. This is a peculiar phenomenon and one reason for this is a weak demand even under a low interest rate environment. In this section, we assume a low intertemporal elasticity of substitution of expenditure, i.e., a low elasticity of the output gap to the real interest rate. Cashin and Unayama (2016) estimate the intertemporal elasticity of substitution in Japan as $\chi = 0.21$. We use this value and other parameters as given in Table 1 for the simulation.

Figure 2 shows the simulation result.[14] Under a low elasticity of the output gap to the real interest rate, the zero interest rate policy continues beyond the fourth quarter of 2024. The model well replicates the output gap data. Our conclusion that a zero interest rate policy continues even after inflation rates exceed over 2 percent still holds.

5.2 Discounted IS Curve

As shown in Del Negro et al. (2023), the impact of forward guidance is too powerful in new Keynesian models. Some previous studies solve this issue, so called the forward guidance puzzle, in models where the expectation term in the IS curve is discounted as shown in McKay et al. (2017); Nakata et al. (2019); Gabaix (2020).

[14] We assume -37.2 percent of the natural rate shock and -0.95 percent of cost-push shock at a time zero as a quarterly base.

Following McKay et al. (2017), Gabaix (2020) and Nakata et al. (2019), we assume a discounted IS curve as follows:

$$x_t = \delta \mathrm{E}_t x_{t+1} - \chi \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n \right), \tag{11}$$

where δ is a parameter for discounting. The first-order condition of equation (7) is replaced by

$$\lambda_x x_t + \phi_{1t} - \delta \beta^{-1} \phi_{1t-1} - \kappa \phi_{2t} = 0, \tag{12}$$

where we set δ to 0.856 following Hirose et al. (2023) with other parameters as given in Table 1.

Figure 3 shows the simulation result.[15] Compared to the case of the simple IS curve in Figure 1, the model's consistency with the output gap data has improved. The overshooting of the output gap is mitigated because the power of commitment is reduced. On the other hand, the duration of the zero interest rate policy is extended beyond the fourth quarter of 2024 due to the weak power of commitment. Our conclusion for a prolonged zero interest rate policy and inflation overshooting does not change.

5.3 Inflation Persistence

In this section, we analyze how the simulation results change by setting γ to 0 (i.e., purely forward-looking), 0.358 (Hirose, 2020), 0.631 (Hirose and Kurozumi, 2012), and 0.862 (Sugo and Ueda, 2008) with other parameters as given in Table 1. For these simulations, we replace π_t by $\pi_t - \bar{\pi}$ in the Phillips curve (2) and the welfare loss function (3) and analyze optimal monetary policy, where $\bar{\pi}$ is exogenously given anchored inflation rate and $\pi_t = \bar{\pi}$ in the steady-state.

Figure 4 shows the simulation result.[16] With the lower values of γ, such as 0 and 0.358, inflation overshooting occurs at the earlier period, and the longer zero interest

[15]We assume -8.9 percent of the natural rate shock and -0.92 percent of cost-push shock at a time zero as a quarterly base.

[16]We assume -16.1, -15.5, 15.1, and -14 percent of the natural rate shock and -1.2, -1.1, -1, and -0.95 percent of cost-push shock at a time zero as a quarterly base for case of $\gamma = 0$, 0.358, 0.631, and 0.862, respectively.

rate well explains these initial rises in inflation rates. There is no exit from the zero interest rate policy in these cases. For the high values of γ, such as 0.631 and 0.862, a significant inflation-overshooting occurs toward the end of the period, and the lift-off from the zero interest rate is shown. This is because, as shown by Michau (2019) and Hasui et al. (2024), the commitment policy can become more front-loaded as inflation persistence increases. Among the four cases, the case of $\gamma = 0.862$ is the best to replicate inflation rates. The most important result is that the zero interest rate policy continues even after inflation rates reach 2 percent and are stably over it for all cases.

5.4 Interest-Rate Smoothing

To describe a gradual lift-off from the zero interest rate, we introduce the squared term of the interest rate difference into the welfare loss function as follows[17]:

$$\mathrm{L}_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda_x x_t^2 + \lambda_\Delta(i_t - i_{t-1})^2, \tag{13}$$

where we set $\lambda_\Delta = 0.0137$ with other parameters as given in Table 1.[18] The first-order conditions (8), (9), and (10) are replaced with the following equations:

$$i_t[\lambda_\Delta(i_t - i_{t-1}) - \beta\lambda_\Delta(\mathrm{E}_t i_{t+1} - i_t) + \chi\phi_{1t}] = 0, \tag{14}$$

$$\lambda_\Delta(i_t - i_{t-1}) - \beta\lambda_\Delta(\mathrm{E}_t i_{t+1} - i_t) + \chi\phi_{1t} \geq 0, \tag{15}$$

$$i_t \geq 0. \tag{16}$$

Figure 5 shows the simulation result.[19] According to the figure, the nominal interest rate lift-off from zero is more gradual in comparison to the case without the squared term

[17]Kobayashi (2008) shows that a central bank has an incentive to smooth the policy rate under an imperfect financial market in which the lending rate is sticky. Woodford (2003b) uses the objective function (13) under the discretionary equilibrium to achieve optimal monetary policy as a delegation problem.

[18]λ_Δ consists of various deep parameters. We set deep parameters following Woodford (2003a) except for those in the present paper.

[19]We assume -13 percent of the natural rate shock and -0.96 percent of cost-push shock at a time zero as a quarterly base.

of the interest rate difference in Figure 1. Although the lift-off from the zero interest rate happens two periods earlier when compared to Figure 1, it is confirmed that the prolonged zero interest rate policy remains in place. The policy rate is still as low as one percent in the last period of the simulation. As a result, the responses of inflation and the output gap do not significantly differ from Figure 1.

5.5 Simple Interest Rate Rule

A prolonged zero interest rate policy is a feature of the commitment policy. However, it is not necessarily obvious whether the long zero interest rate policy under commitment causes inflation overshooting in our analysis. In this section, we assume a simple interest rate rule as follows to investigate whether inflation overshooting occurs:

$$i_t = \max\left[0, i^* + \phi_\pi(\pi_t - \bar{\pi})\right], \tag{17}$$

where we set $\phi_\pi = 5$ following Fujiwara et al. (2013). This rule does not incorporate the commitment policy by history dependence. Other parameters are given in Table 1.

Figure 6 shows the simulation result.[20] Regardless of when the zero interest rate policy is terminated, there is no inflation overshooting exceeding 2 percent. The result shows that the prolonged zero interest rate policy under commitment makes inflation overshooting over 2 percent.

5.6 Inflation Targeting

Under the inflation target policy, a mandate for a central bank is to stabilize inflation rates around the target level such as 2 percent. Here, we simply assume a welfare loss function including inflation deviation from the target level rather than the welfare loss function (3) as

$$\mathrm{L}_t = (\pi_t - \bar{\pi})^2 + \lambda_x x_t^2, \tag{18}$$

[20]We assume -7.8 percent of the natural rate shock and -0.82 percent of cost-push shock at a time zero as a quarterly base.

where $\bar{\pi}$ is set to 2 percent annually. Other parameters are given in Table 1. The first-order condition (6) is replaced with the following equation:

$$(\pi_t - \bar{\pi}) - \beta^{-1}\chi\phi_{1t-1} - \beta\gamma \mathrm{E}_t\phi_{2t+1} + (\beta\gamma + 1)\,\phi_{2t} - \phi_{2t-1} = 0. \tag{19}$$

Figure 7 shows the simulation result.[21] According to the figure, the increase in the output gap is larger compared to the case of optimal monetary policy in Figure 1. This is because, with the removal of the inflation persistence term from the welfare loss function, the timing of lift-off from the zero interest rate is delayed. As a result, the demand is overstimulated.

5.7 Alternative Scenarios for Natural Rates and Anchored Inflation Expectations

5.7.1 High State Economy: 2.5 Percent Nominal Interest Rate

We assume that an inflation expectation is anchored at 2 percent and the natural interest rate is given by the upper bound of the estimation as 0.5 percent. In this case, the nominal interest rate in the steady-state for simulation is 2.5 percent and we set $\beta = 0.9938$. Other parameters are given in Table 1.

Figure 8 shows the simulation result.[22] A clear difference from Figure 1 is three quarters earlier termination of the zero interest rate policy. A reason for this is that the monetary easing becomes stronger for the same zero interest rate policy as the nominal interest rate in the steady-state becomes higher by a higher anchored inflation expectation and a higher natural interest rate, as described in equation (1).

[21] We assume -15 percent of the natural rate shock and -0.62 percent of cost-push shock at a time zero as a quarterly base.

[22] We assume -15 percent of the natural rate shock and -1.1 percent of cost-push shock at a time zero as a quarterly base.

5.7.2 Low State Economy: 0.5 Percent Nominal Interest Rate

We assume that an inflation expectation is anchored at 1.5 percent and the natural interest rate is given by the lower bound of estimation as -1 percent. In this case, the nominal interest rate in the steady-state is given by 0.5 percent and we set $\beta = 0.9987$. Other parameters are given in Table 1.

Figure 9 shows the simulation result.[23] A clear difference from Figure 1 is that the zero interest rate policy continues one quarter longer.

5.8 An Alternative Specification for Phillips Curve

Our analyses are based on a new Keynesian model that incorporates the price indexation hypothesis. One of the reasons for this is that many previous studies, such as Christiano et al. (2005), Smets and Wouters (2003, 2007), and Sugo and Ueda (2008), explain inflation persistence through the price indexation hypothesis. However, there are various studies with alternative explanations for inflation persistence, such as Rule-of-Thumb (Galí and Gertler, 1999; Amato and Laubach, 2003) and information stickiness (Mankiw and Reis, 2002). These studies also explain inflation persistence by explicitly including lagged inflation rates in the Phillips curve.[24]

In this section, we use the Rule-of-Thumb hypothesis model of Amato and Laubach (2003). In this case, the Phillips curve and period-loss function are given as follows:

$$\pi_t = \tilde{\kappa} x_t + \varphi_b \pi_{t-1} + \varphi_f \mathrm{E}_t \pi_{t+1}, \tag{20}$$

$$\mathrm{L}_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_{\Delta\pi} (\pi_t - \pi_{t-1})^2, \tag{21}$$

[23]We assume -12 percent of the natural rate shock and -0.95 percent of cost-push shock at a time zero as a quarterly base.

[24]Among others, Sheedy (2010) incorporates a hazard function into the model, assuming that the longer fixed prices are more likely to be reset, and explains inflation persistence. More recently, Kurozumi and Van Zandweghe (2023) introduce a Kimball-type non-CES aggregator and trend inflation in a model to theoretically demonstrate intrinsic inflation persistence.

where

$$
\begin{aligned}
\tilde{\kappa} &= \frac{\vartheta\alpha}{\alpha + (1-\vartheta)[1-\alpha(1-\beta)]}\kappa, \\
\varphi_b &= \frac{1-\vartheta}{\alpha + (1-\vartheta)[1-\alpha(1-\beta)]}, \\
\varphi_f &= \frac{\alpha\beta}{\alpha + (1-\vartheta)[1-\alpha(1-\beta)]}, \\
\lambda_{\Delta\pi} &= \frac{1-\vartheta}{\alpha\vartheta},
\end{aligned}
$$

and $1-\vartheta$ denotes the probability that firms set prices according to the Rule-of-Thumb. The first-order conditions for the optimal monetary policy under commitment are given as follows:

$$\pi_t + \lambda_{\Delta\pi}(\pi_t - \pi_{t-1}) - \beta\lambda_{\Delta\pi}\left(\mathrm{E}_t\pi_{t+1} - \pi_t\right) - \beta^{-1}\chi\phi_{1t-1} - \frac{\varphi_f}{\beta}\phi_{2t-1} - \beta\varphi_b\mathrm{E}_t\phi_{2t+1} + \phi_{2t} = 0, \tag{22}$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \tilde{\kappa}\phi_{2t} = 0, \tag{23}$$

$$i_t\phi_{1t} = 0, \tag{24}$$

$$\phi_{1t} \geq 0, \tag{25}$$

$$i_t \geq 0, \tag{26}$$

where ϕ_1 and ϕ_2 are the Lagrange multipliers associated with the IS constraint and the Phillips curve constraint, respectively.

We set ϑ to 0.6, following Amato and Laubach (2003) for the US economy. Other parameters are given in Table 1. Thus, φ_b is set as 0.3139. Eventually, this is a similar setting in the Phillips curve (2) with $\gamma = 0.631$ of Hirose and Kurozumi (2012) for a coefficient of the lagged inflation rate, and the simulation result is also similar.

Figure 10 shows the simulation result.[25] According to the figure, inflation rates rise to 2 percent and stay around 2 percent. In particular, the model is well suited to explain

[25]We assume -16.1 percent of the natural rate shock and -0.7 percent of cost-push shock at a time zero as a quarterly base.

the initial inflation surge. The zero interest rate policy prolongs even after the inflation rate rises over 2 percent.

6 Concluding Remarks

After the zero interest rate policy for a few decades, the BOJ now faces the exit policy from a liquidity trap. We evaluate whether the BOJ's monetary policy is optimal monetary policy or not by using the conventional new Keynesian model.

We show that optimal monetary policy in a liquidity trap well replicates the BOJ's monetary policy and the Japanese economy during and after the pandemic period. The BOJ's monetary policy has several similarities with optimal monetary policy in a liquidity trap. The zero interest rate policy continues even after inflation rates sufficiently exceed 2 percent. Optimal monetary policy continues the zero interest rate policy until the second quarter of 2024. The BOJ continues the zero interest rate policy at least until the second quarter of 2024. Recent high inflation rates can be explained by a prolonged zero interest rate policy under the commitment.

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Table 1: Naïve Calibration for Japan

Parameters	Values	Explanation
β	0.9975	Discount Factor
χ	0.646	Intertemporal Elasticity of Substitution of Expenditure
ω	2.149	Elasticity of Firm's Real Marginal Cost
θ	6	Elasticity of Substitution across Goods
κ	0.0048	Elasticity of Inflation to Output Gap
α	0.875	Price Stickiness
γ	1	Degree of Inflation Persistence
λ_x	0.0008	Weight for Output Gap
i^*	1	Steady-state Nominal Interest Rate (Annual)

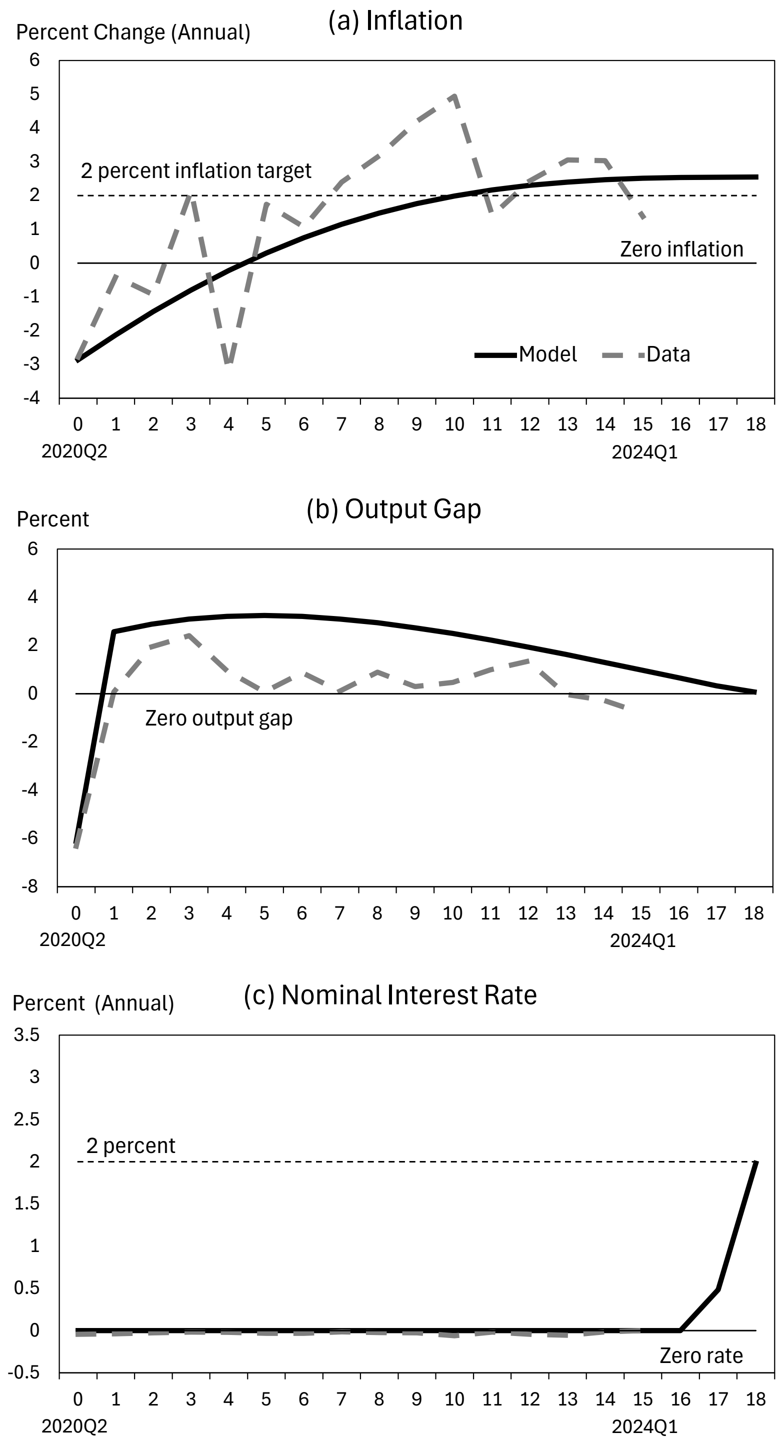

Figure 1: Simulation for Japanese Monetary Policy

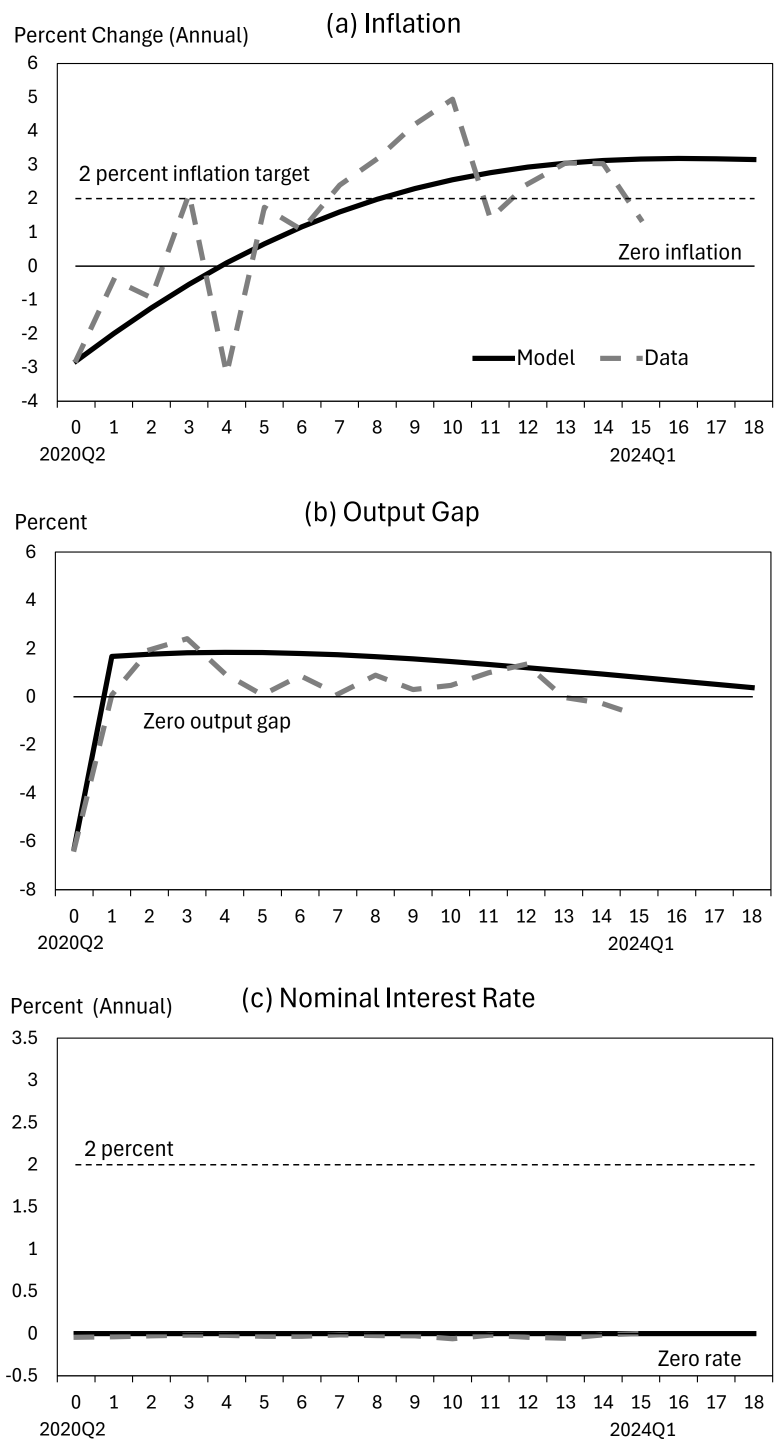

Figure 2: Simulation for Japanese Monetary Policy: Low Elasticity of Demand to Real Interest Rate (Cashin and Unayama, 2016).

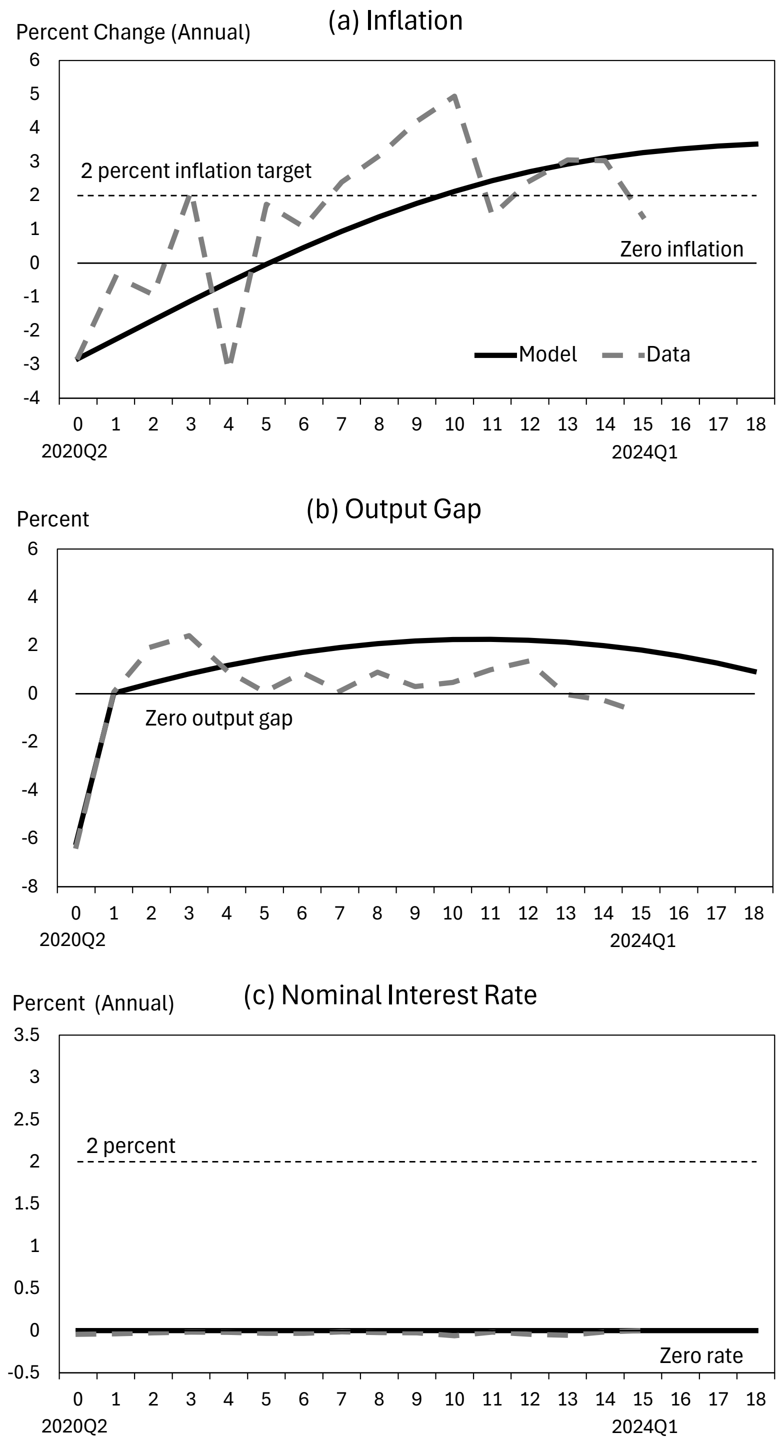

Figure 3: Simulation for Japanese Monetary Policy: Discounted IS Curve

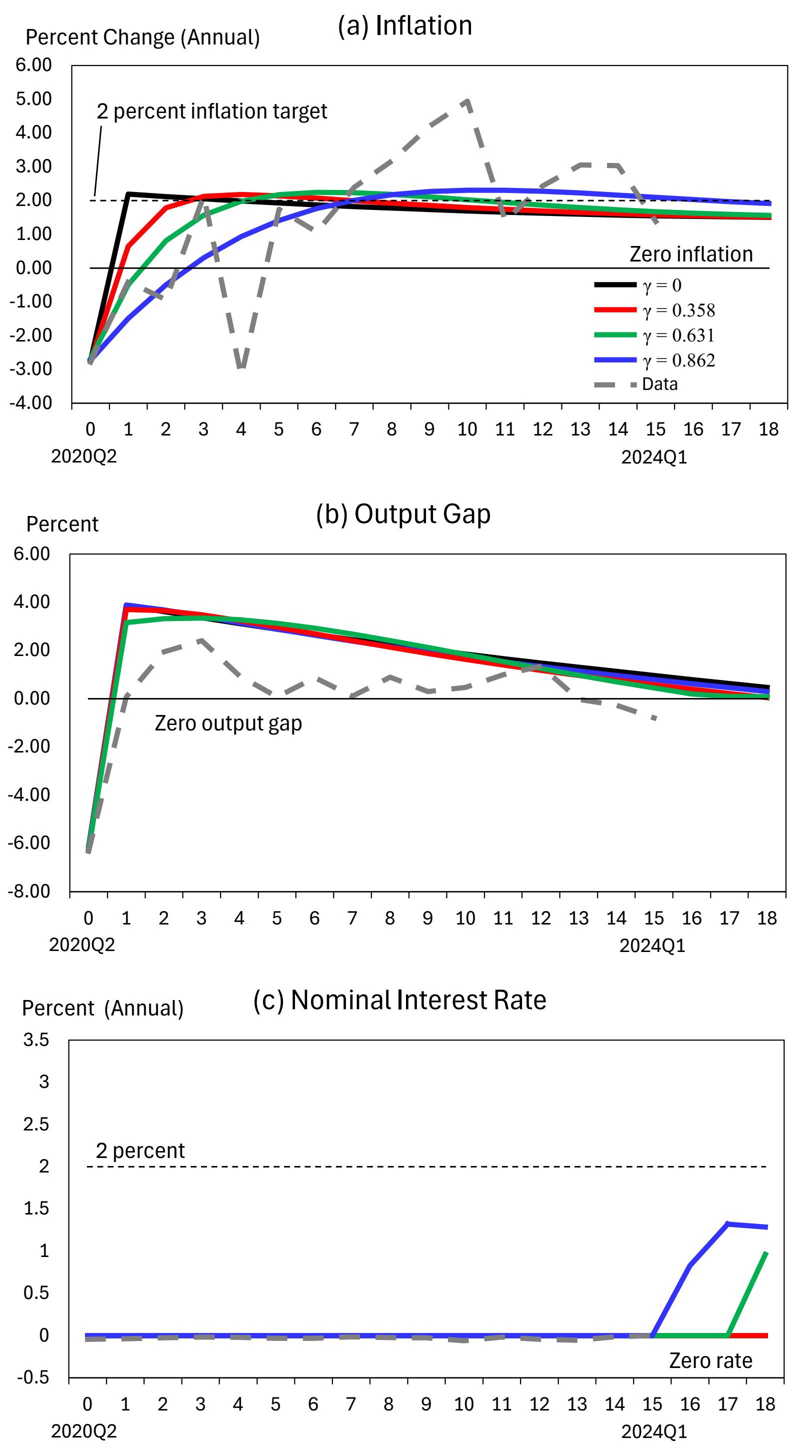

Figure 4: Simulation for Japanese Monetary Policy: Alternative γ.

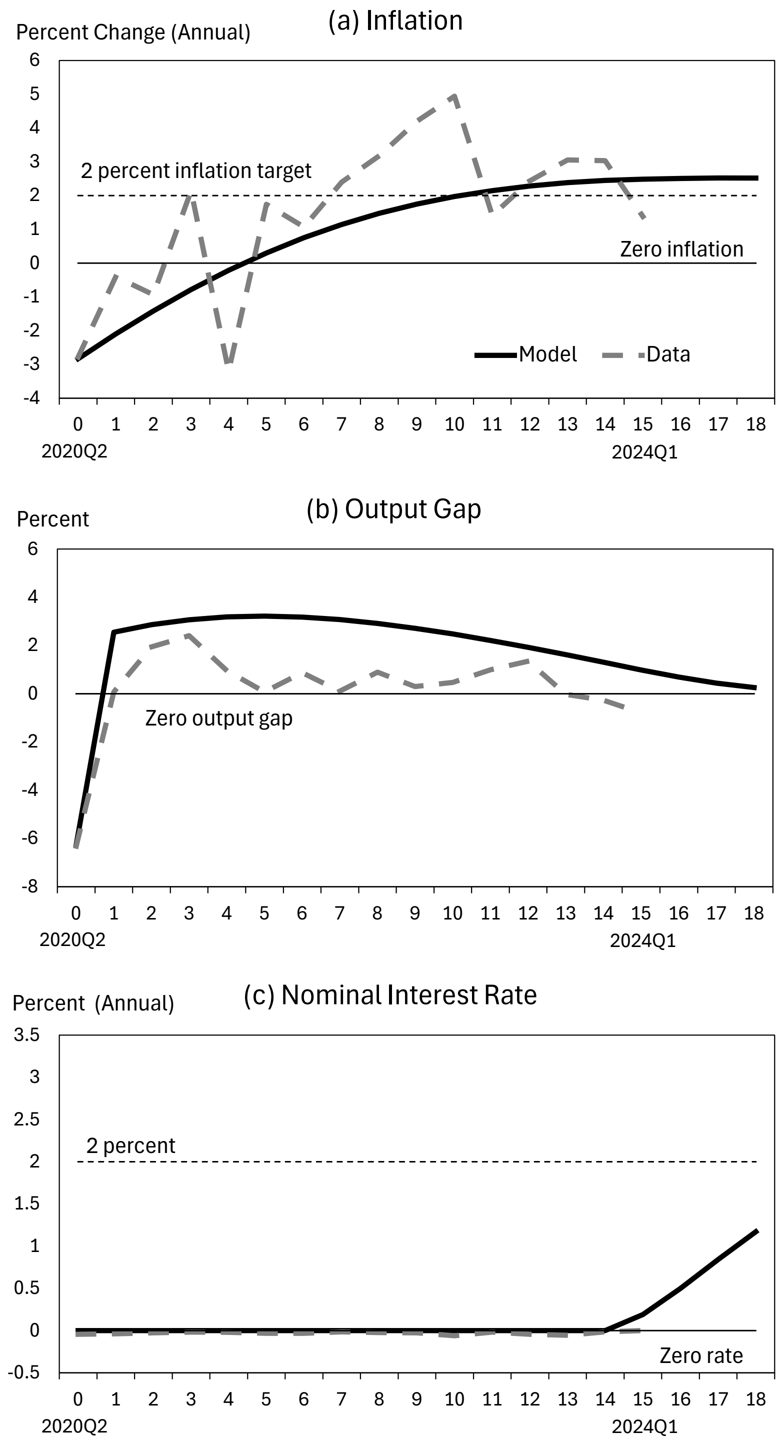

Figure 5: Simulation for Japanese Monetary Policy: Interest-Rate Smoothing

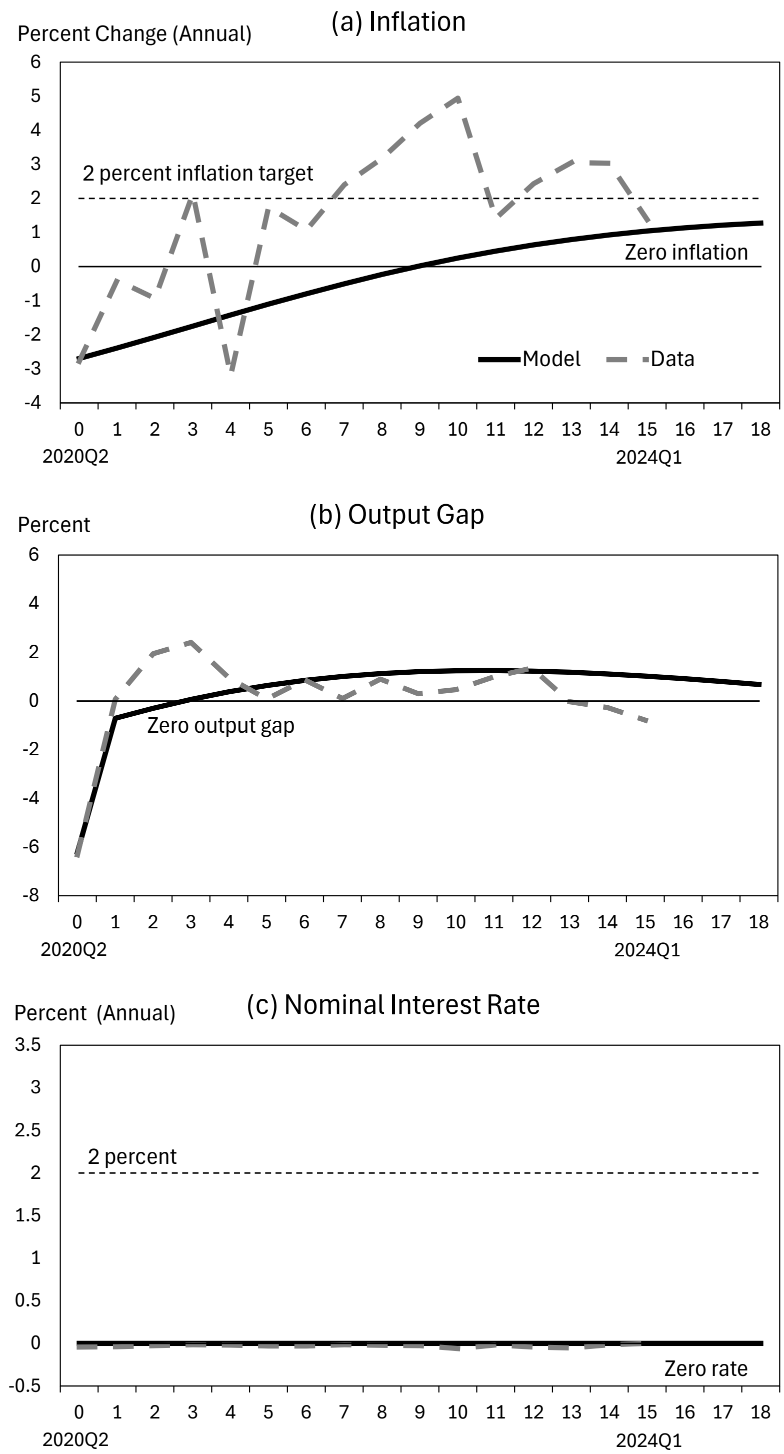

Figure 6: Simulation for Japanese Monetary Policy: Simple Interest Rate Rule

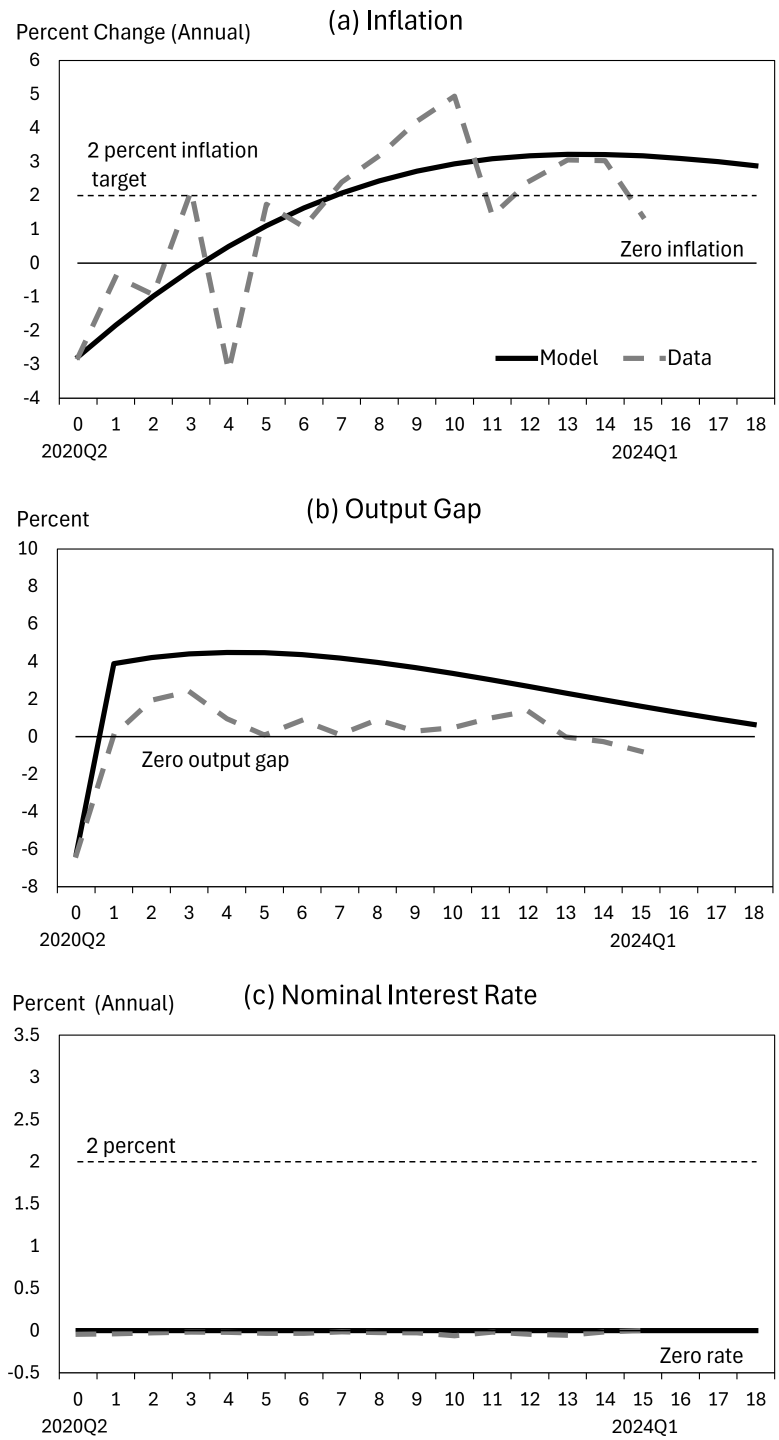

Figure 7: Simulation for Japanese Monetary Policy: Inflation Target

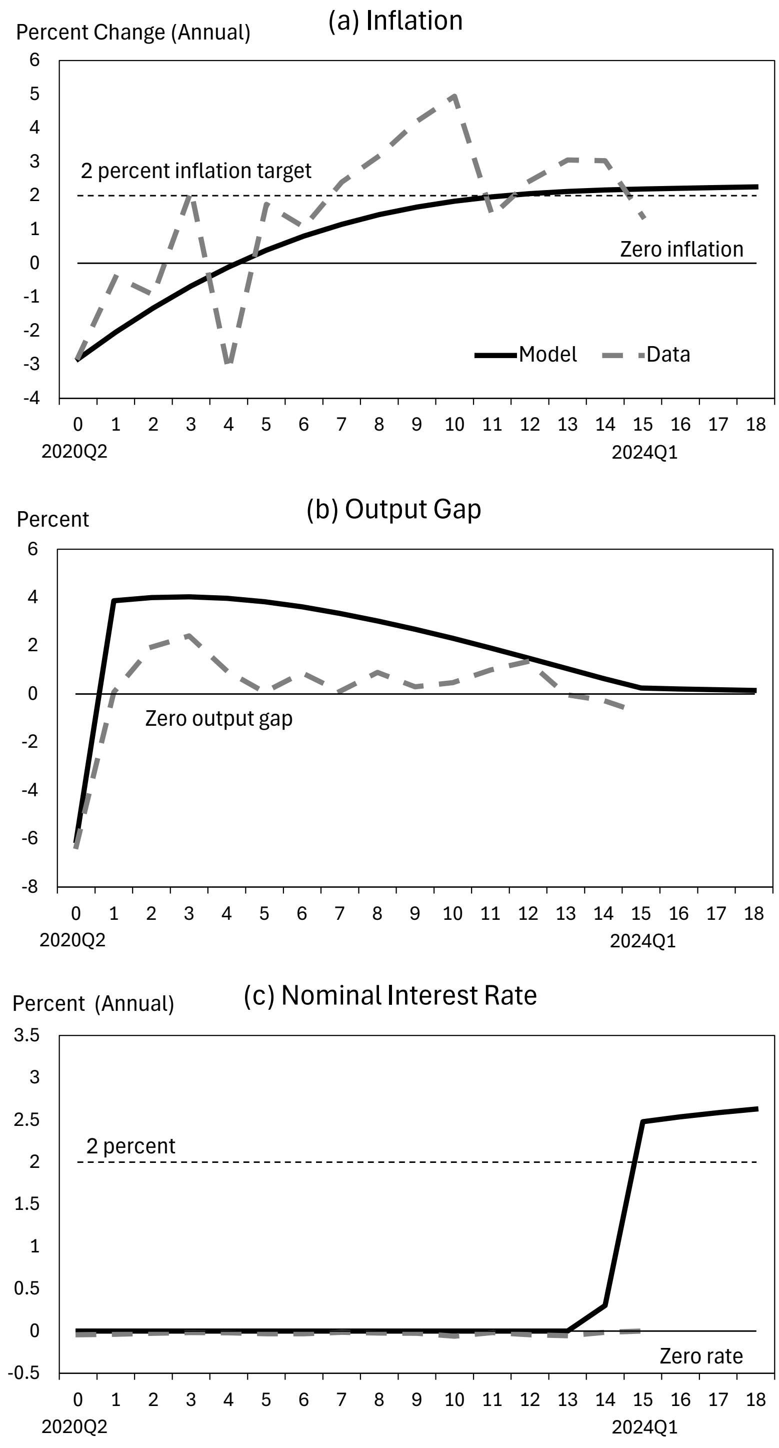

Figure 8: Simulation for Japanese Monetary Policy: High State Economy

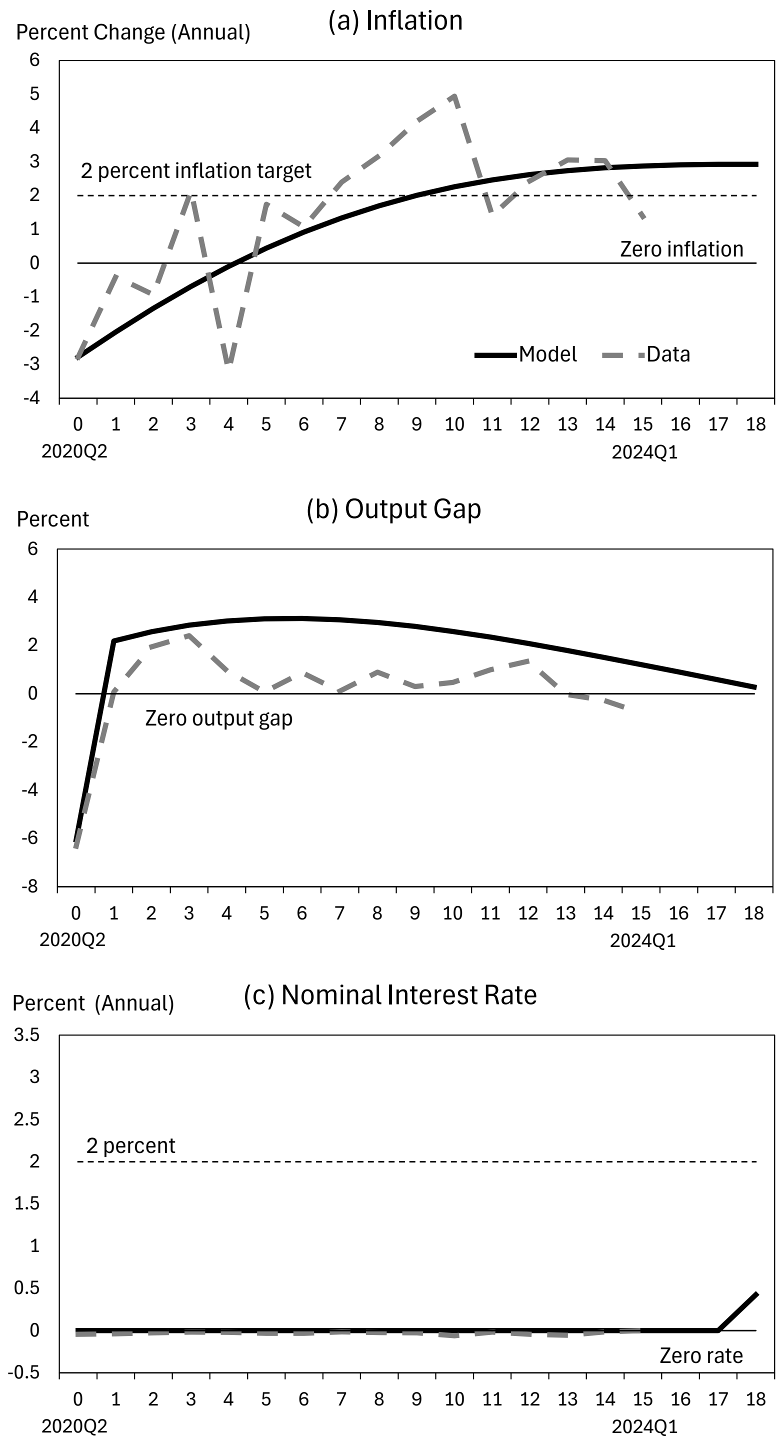

Figure 9: Simulation for Japanese Monetary Policy: Low State Economy

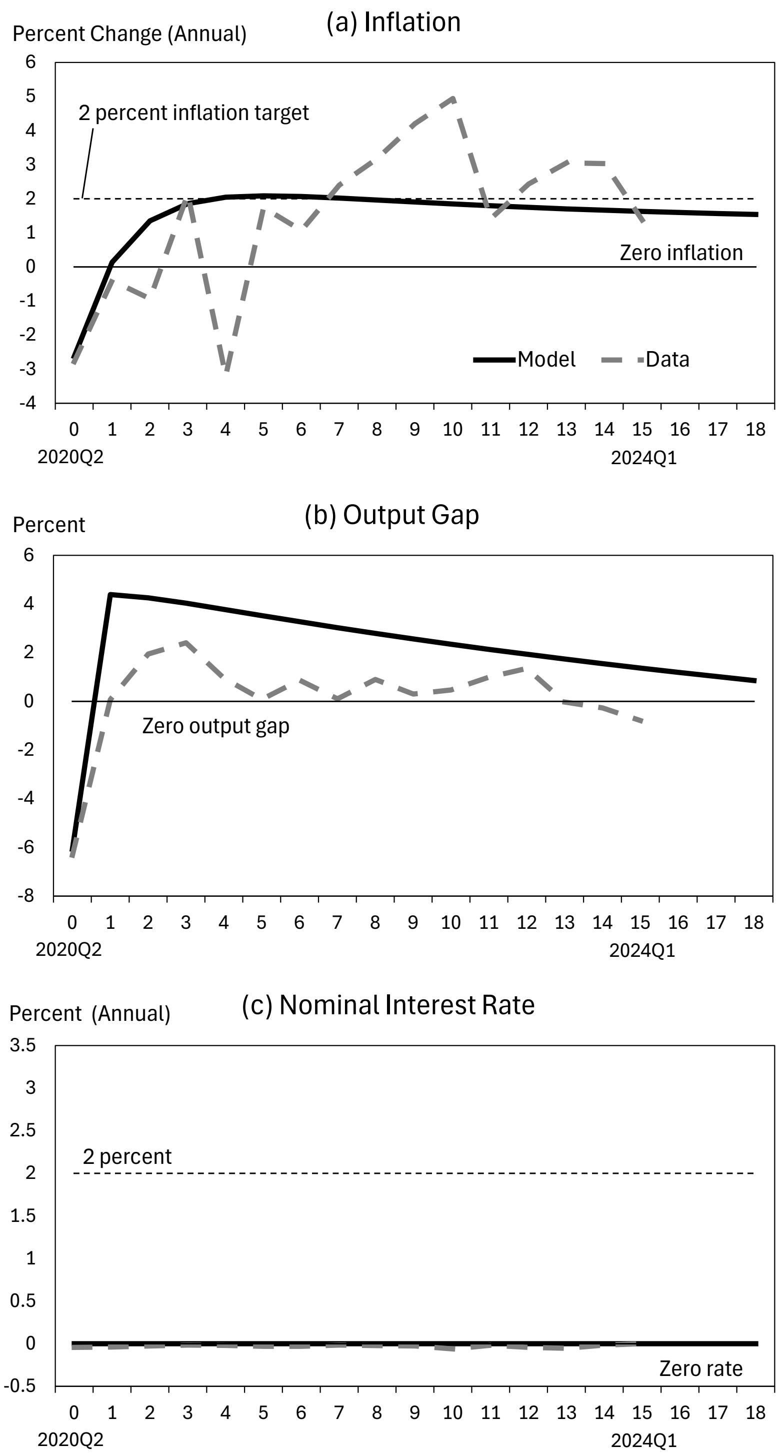

Figure 10: Simulation for Japanese Monetary Policy: Amato and Laubach (2003).