Mini Course on Structural Estimation of Static and Dynamic Games

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Part III: Estimation of Dynamic Games

Introduction

- Firms often compete each other overtime
- e.g. chain stores trying expand their network
- One's current decision affect its own current and future profit
  - by its decision itself (direct effect)
  - by affecting its own future decision (indirect)
  - by affecting its rivals' current and future decisions (indirect)
- Needs to construct a dynamic game

Estimation of Dynamic Game

- Dynamic game contains
  - strategic interaction between agents
  - dynamic optimization

- Straightforward estimation (nested fixed point maximum likelihood) is virtually impossible (don't even think about it!)

- Need to rely on two-step methods:
  - Nested pseudo likelihood (NPL) by Aguirregabiria and Mira (2007)
  - Another two-step method or BBL by Bajari, Benkard and Levin (2008)

Recent Research

- Ryan (2009), MIT WP: Studies the impacts of environmental regulation on entry/investment decisions of U.S. Portland cement industry

- Sweeting (2009), Duke WP: Studies whether radio stations reposition their products to deter future entries

- Snider (2009), Minnesota WP: Studies whether airline companies are engaged in predatory behavior

Example: Competition between Hotel Chains

- Suzuki (2009) considers a dynamic entry-exit game between hotel chains to recover the impacts of land use regulation on the cost structure of hotel chains
- Assume each local market is isolated
- Hotel chains decide the number of hotels they operate every period
- Hotel chains incur
  - one-shot sunk entry cost to open a new hotel
  - sunk entry cost is random and private

Empirical Structure

- The data
  - the number of hotels each chain operates for a certain period
  - each hotel's revenue
  - market characteristics
- By using these data, want to recover the cost parameters that rationalize both entry/exit behavior and revenue data

Idea of Identification

- Suppose you observe a market with one firm overtime
- This observation suggests this market is large enough to support one hotel but not more
- Revenue data helps to identify cost level

The Model: Basic

- Common state space: $s_t = (h_t, x_t)$
- Choice variable:
  $a_{it} \in A(h_{it}) = \{-2, -1, 0, 1, 2\} \cap \{0 \leq h_{it} + a_{it} \leq 7\}$
- Notations:

$$
\begin{array}{rcl}
i & \in & \{1, \ldots, N\}: \text{ hotel chain} \\
t & \in & \{1, 2, \ldots, \infty\}: \text{ period} \\
h_{it} & : & \# \text{ of hotels chain } i \text{ operates at } t \\
x_t & : & \text{market characteristics} \\
a_{it} & : & \# \text{ of hotels chain } i \text{ opens/closes at } t
\end{array}
$$

The Model: Period Profit

- Period profit function:

$$\begin{aligned}\pi_i\left(a_{it}, s_t, v_{it}\right) &= R\left(s_t\right) - \delta_i h_{it} - \left(e_i + \rho_i v_{it}\right) 1\left(a_{it} > 0\right) a_{it} \\ &= \Psi\left(a_{it}, s_t, v_{it}\right)' \theta_i\end{aligned}$$

where

$$\begin{aligned}\Psi\left(a_{it}, s_t, v_{it}\right) &= \begin{bmatrix} R_i\left(s_t\right) \\ -h_{it} \\ -1\left(a_{it} > 0\right) a_{it} \\ -1\left(a_{it} > 0\right) a_{it} v_{it} \end{bmatrix} \\ \theta_i &= \left[1, \delta_i, e_i, \rho_i\right]\end{aligned}$$

- Notations:

δ_i: operating cost

e_i: the mean of entry cost

ρ_i: the stddev of entry cost

$v_{it} \sim N\left(0, 1\right)$

The Model: Value Function

$$
\begin{aligned}
V_i\left(s_t;\sigma_i,\sigma_{-i}\right) &= E\left[\sum_{\tau=t}^{\infty}\beta^{\tau-t}\Psi\left(\sigma_i\left(s_\tau,v_{i_\tau}\right),s_\tau,v_{i_\tau}\right)'\theta_i\middle|\sigma_{-i}\right] \\
&= W_i\left(s_t;\sigma\right)'\theta_i
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_i\left(s_t,v_i\right) &: \quad \text{policy function } \left(\sigma_i : S\times\mathbb{R}\rightarrow A\right) \\
W_i\left(s_t;\sigma\right) &= E\left[\sum_{\tau=t}^{\infty}\beta^{\tau-t}\Psi\left(\sigma_i\left(s_\tau,v_{i\tau}\right),s_\tau,v_{i\tau}\right)\middle|\sigma_{-i}\right]
\end{aligned}
$$

The Model: Markov Perfect Equilibrium

- In a Markov perfect equilibrium, an equilibrium strategy $\sigma_i^*(s, v)$ must be the best response to its rivals' equilibrium strategy σ_{-i}^*:

$$V_i\left(s; \sigma_i^*, \sigma_{-i}^*\right) \geq V_i\left(s; \sigma_i', \sigma_{-i}^*\right)$$

for all $i, s \in S$ and σ_i'

- Exploiting the linearity of the period profit function,

$$\left\{W_i\left(s; \sigma_i^*, \sigma_{-i}^*\right) - W_i\left(s; \sigma_i', \sigma_{-i}^*\right)\right\} \theta_i \geq 0$$

for all $i, s \in S$ and σ_i'

Nested Fixed Point Algorithm

- Brute-force method is hopeless in most dynamic games
- Evaluating the likelihood function even once could take more than a day
- Plus, multiple equilibria are prevalent

Using BBL

- Suzuki (2009) used BBL for estimating structural cost parameters
- It does not require
  - discretization of continuous state variables
  - calculating the inverse matrix
- Three continuous variables with 20 intervals would be a $20^3 \times 20^3$ matrix

Estimation Step 1: Revenue Function Estimation

$$\ln r_{ikt} = \gamma_i + \eta_1 + x_t' \eta_2 - \eta_3 \ln \left(\Sigma_j h_{jt} \right) - \eta_4 \ln h_{it} + \epsilon_{ikt}$$

- Exploit the panel structure
- Market specific fixed effects (at least partially) take care of the endogeneity of h_{it}
- Chain-specific effects are also taken into account
- The transition function (population, establishments and trend) are estimated by AR1 regression

Estimation Step 2: Policy Function Estimation

$$
\begin{aligned}
y_{it}^{*} \;=\; & \alpha_i + \alpha_m + \alpha_2 \ln x_t - \alpha_3 \ln h_{it} - \alpha_4 \left(\ln h_{it}\right)^2 \\
& -\alpha_5 \left(\ln\left(\Sigma_j h_{jt}\right)\right) - \alpha_6 \left(\ln\left(\Sigma_j h_{jt}\right)\right)^2 + \omega_{it}
\end{aligned}
$$

- Estimate the policy function by using an ordered logit
- Use market-dummy to take into account the impact of unobservable market-specific characteristics on their entry-exit decision
- Take into account the change in choice set

Estimation Step 3: Recovering Cost Parameters

- Recover chain i's structural cost parameters $\hat{\theta}_i$ that rationalizes the estimated revenue function and the estimated policy function
- Each chain's policy function must be the best response to its rivals' policy functions
- The observed policy must beat other possible policies when its rivals' play observed policies
- BBL shows that we can exploit this property to identify structural parameters
- No data are used once policy functions are estimated

Implementing BBL Step by Step

- Focus on chain i's structural parameters in market m
- Generate chain i's fake policies $\{\sigma_i^m\}_{m=1}^{NI}$ by slightly perturbing the observed policy function
  - How we should make perturbations is an open question
  - Different perturbation will affect the efficiency of estimators
  - Might make a sense to consider a perturbation with clear intuition (e.g., never enter)

Implementing BBL Step by Step

- Want to approximate the expected sum of future profit when they follow certain strategies
- Assuming other chains follow the observed policy $\hat{\sigma}_{j \neq i}$, simulate the model and calculate $W_i(s_t; \sigma)$ for the observed policy $\hat{\sigma}_i$ as well as σ_i^m
- Simulate economy for T periods
- T should be large enough so that β^T is sufficiently small
- The number of simulation should be very large since it needs to capture entry/exit behavior

Exploiting Linearity

- Note that linearity assumption allows us to separate simulation from estimation
- Simulation calculates $W_i\left(s_t;\sigma\right)$
- No structural parameters are directly involved
- Otherwise, need to conduct simulations for each θ

Computational Tips in BBL

- Under linearity, you can separate simulation from optimization
- BBL requires running a large number of simulations
- Good thing is each simulation is independent
- Can save time by using multiple processors/computers at the same time
- Matlab has a toolbox for parallel computation

Implementing BBL Step by Step

- Try to find the set of parameters that minimizes the following loss function:

$$
\begin{aligned}
\theta_i^* &= \arg\min_{\theta} \sum_{m=1}^{NI} \left(\min\left\{g_m(\theta), 0\right\}\right)^2 \\
g_m(\theta) &= \left\{W_i\left(s; \sigma_i^*, \sigma_{-i}^*\right) - W_i\left(s; \sigma_i', \sigma_{-i}^*\right)\right\}\theta_i
\end{aligned}
$$

- When the observed policy beats a fake policy, it adds zero to the loss function

- When a fake policy beats the observed policy, it adds squared profit-difference to the loss function

Conducting Counterfactual Experiments

- A motivation of doing structural estimation is to conduct counterfactual experiments
- Using structural parameters, the model can calculate an equilibrium under alternative policies
- Can compare two competing policies by comparing the welfare level they bring

Conducting Counterfactual Experiments

- Note that in counterfactual experiments, you need to solve the model explicitly
- In complicated models, solving an equilibrium might be virtually impossible
- You might need to make a model simple in simulations
- Also multiple equilibria are prevalent in dynamic games
- The impacts of policies might not be pinned down

Summary

- Go over the very basics of estimating a dynamics entry-exit model
- Nested fixed point algorithm turns out to be virtually impossible
- BBL with linearity assumption significantly reduces computational burden without hurting consistency